7 Properties of Exponents

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7.1 Exponentially Speaking

Powers and Exponents

Learning Goals

In this lesson, you will:

- Expand a power into a product.
- Write a product as a power.
- Simplify expressions containing integer exponents.

Key Terms

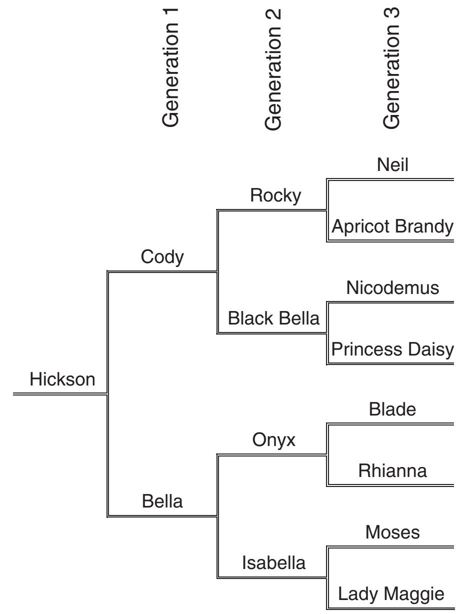
- power
- base
- exponent

She was more than "man's best friend." She was also many, many sightless people's best friend, too. Buddy was a German shepherd who is considered to be the first Seeing Eye dog in the United States. In 1927, Morris Frank, a sightless person, traveled to Switzerland to train with a Seeing Eye dog and later returned to the United States to help start the first school to train Seeing Eye dogs. Since then, thousands of Seeing Eye dogs and sightless people have been matched together.

Do you think that Seeing Eye dogs' mothers and fathers were Seeing Eye dogs as well? If yes, how many generations of Seeing Eye dogs do you think there are?

Problem 1 Prime Factorization and Exponents

Jake purchased an English Mastiff puppy that he named Hickson. The breeder provided documentation that verified Hickson's lineage for three generations as shown.



A dog's lineage is similar to a person's family tree. It shows a dog's parents, grandparents, and great-grandparents.

1. How many parents does Hickson have? What are his parents' names?

a. How many grandparents does Hickson have?

b. How many great-grandparents does Hickson have?

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c. Do you see a pattern between each generation of Hickson's lineage?

2. Jake wants to trace Hickson's lineage back seven generations. How many sires (male parents) and dams (female parents) are there in seven generations of Hickson's lineage? Complete the second column of the table to show the total number of dogs in each generation.

	Number of Sires and Dams		
Generation 1			
Generation 2			
Generation 3			
Generation 4			
Generation 5			
Generation 6			
Generation 7			

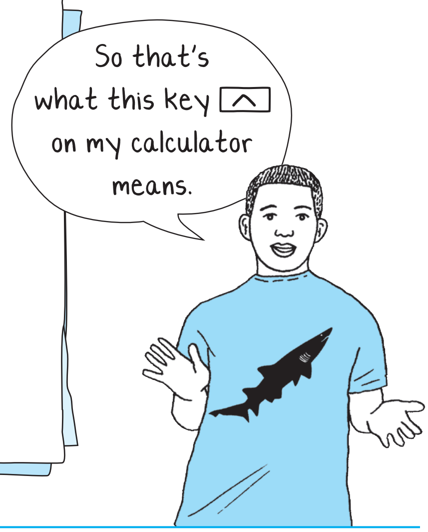
An expression used to represent the product of a repeated multiplication is a *power*. A **power** has a *base* and an *exponent*. The **base** of a power is the expression that is used as a factor in the repeated multiplication. The **exponent** of a power is the number of times that the base is used as a factor in the repeated multiplication.

You can write a power as a product by writing out the repeated multiplication.

$$2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

The power 2^7 can be read as:

- "two to the seventh power."
- "the seventh power of two."
- "two raised to the seventh power."



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3. In the third column of the table, write each generation total as a product. Don't forget to name the column.
4. In the fourth column of the table, write each generation total as a power. Don't forget to name the column.
5. How many dogs are in Hickson's lineage in just the 12th generation back? Write your answer as a power, and then use a calculator to determine the total number of dogs.

6. How many total sires and dams are there in all three generations shown in Hickson's lineage? Explain your calculation.

Problem 2 Working With Powers

1. Identify the base(s) and exponent(s) in each. Then, write each power as a product. Finally, evaluate the expression.

   a. 5^3

   b. $(-9)^5$

   c. -11^3

   d. $(4)^5(3)^6$

2. Identify the base(s) and exponent(s) in each. Then, write each power as a product.

a. 4^2x^2

b. $(4x)^2$

c. $(-4x)^2$

d. $-(4x)^2$

3. Write each as a product. Then, calculate the product.

a. -1^2

b. -1^3

c. -1^4

d. -1^5

e. $(-1)^2$

f. $(-1)^3$

g. $(-1)^4$

h. $(-1)^5$

4. Identify the base of each power in Question 3, parts (a) through (d), and the base of each power in Question 3, parts (e) through (h).

5. What conclusion can you make about a negative number raised to an odd power?

6. What conclusion can you make about a negative number raised to an even power?

Talk the Talk

1. Write each as a power or a product of powers.

   a. $8 \cdot 8 \cdot 8 \cdot 8$

   b. $-c \cdot c \cdot c \cdot c \cdot c$

   c. $(-m)(-m)(-m)(-m)(p)(p)$

   d. $7 \cdot 7 \cdot 7 \cdot r \cdot r \cdot z \cdot z \cdot z \cdot z$

2. What is the difference between a base and an exponent?

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Be prepared to share your solutions and methods.

7.2 Digital Storage

Multiplying and Dividing Powers

Learning Goals

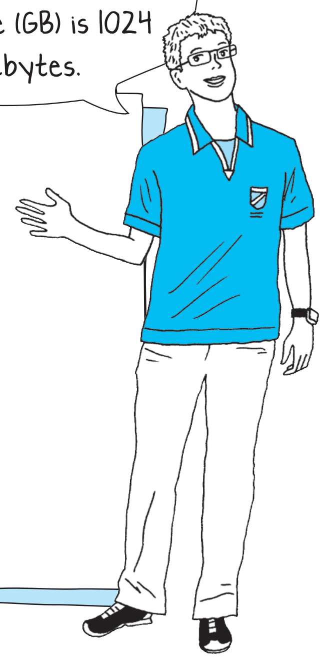
In this lesson, you will:

- Develop a rule to simplify a product of powers.
- Develop a rule to simplify a power of a power.
- Develop a rule to simplify a quotient of powers

Electronic books, or eBooks, have had a much shorter history than printed books, but like most of technology, they have changed rapidly. In 1971, Michael Hart launched the *Gutenberg Project,* making works of literature available in digital form for free. In 1998, the first eBook readers were launched, and in 2000, famous horror-story writer Stephen King authored a book available only in electronic form. Today, people have the ability to download books on eBook readers or cell phones—and not just the text, but also the images. Can you think of features eBooks might offer in the future?

Problem 1 eBooks

File sizes of eBooks, podcasts, and song downloads depend on the complexity of the content and the number of images.



Suppose that a medium-sized eBook contains about 1 megabyte (MB) of information.

Since 1 megabyte is 1024 kilobytes, and 1 kilobyte is 1024 bytes, you can multiply to determine the number of bytes in the eBook:

1 MB = (1024)(1024) = 1,048,576 bytes

There are 1,048,576 bytes in the eBook.

1. Use the method shown in the example to calculate each. One model of an electronic book reader can store up to 256 MB of data.

   a. Calculate the number of bytes the electronic book reader can store.

   256 × __________ × __________

   b. A USB jump drive can hold 2 GB of storage. How many bytes can the USB jump drive hold?

   2 × __________ × __________ × __________

   c. How many times more storage space does the jump drive have than the electronic book reader? Show your work.

   $$\frac{2 \times \underline{\qquad} \times \underline{\qquad} \times \underline{\qquad}}{256 \times \underline{\qquad} \times \underline{\qquad}} = \underline{\qquad} \times \underline{\qquad} = \underline{\qquad} =$$

Computers use binary math, or the base 2 system, instead of the base 10 system.

Base 10	Base 2
$10^1 = 10$	$2^1 = 2$
$10^2 = 10 \times 10 = 100$	$2^2 = 2 \times 2 = 4$
$10^3 = 10 \times 10 \times 10 = 1000$	$2^3 = 2 \times 2 \times 2 = 8$

2. Revisit Question 1, parts (a) through (c), by writing each factor and then your product or quotient as a power of 2.

a.

b.

c.

3. What do you notice about all the bases in Question 2?

4. In parts (a) and (b) of Question 2, how does the exponent in each product relate to the exponents in the factors?

5. In part (c) of Question 2, how does the exponent in the quotient relate to the exponents in the numerator and denominator?

Problem 2 Exploring Products and Powers

1. Rewrite each expression by writing each power as a product. Record the number of times a base is used as a factor.

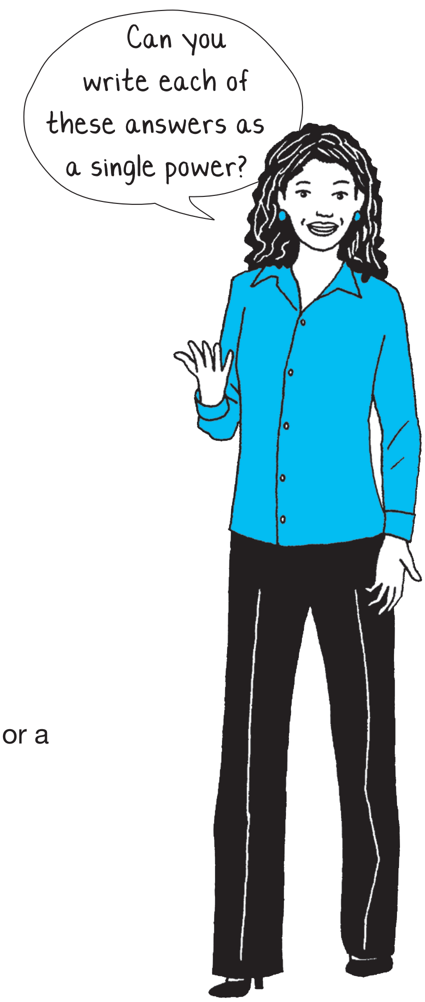
   a. $2^4 \cdot 2^3$

   b. $(-3)^3(-3)^2$

   c. $(a)(a^8)$

   d. $m^2p^2m^3p$

   e. $9^3 \cdot y^2 \cdot 9^2 \cdot y^5$



2. Write each of your answers from Question 1 as a power or a product of powers.

   a.

   b.

   c.

   d.

   e.

3. What relationship do you notice between the exponents in the original expression and the number of factors?

4. Write a rule that you can use to multiply powers.

Problem 3 Exploring a Power to a Power

Sometimes, a power can be raised to a power.

The exponential expression $(4^2)^3$ is a power to a power. It can be written as two repeated multiplication expressions using the definition of a power.

$$(4^2)^3 = (4^2)(4^2)(4^2)$$

$$= (4)(4)(4)(4)(4)(4)$$

There are 6 factors of 4.

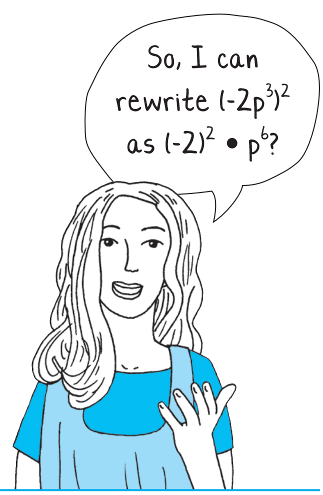
1. Use the definition of a power to write repeated multiplication expressions for each power to a power as shown in the example. Then, record the number of factors.

   a. $(8^2)^3$

   b. $(5^4)^2$

   c. $(j^3)^4$

   d. $(-2p^3)^2$



2. What relationship do you notice between the exponents in each expression in Question 1 and the number of factors?

3. Write a rule that you can use to raise a power to a power.

4. Simplify each expression using the rules you wrote in Questions 2 and 3.

a. $6^4 \cdot 6^3$

b. $x^7 \cdot x^8$

c. $(w^3)^5$

d. $(2ab)^5$

e. $-8h^2 \cdot 5h \cdot 2h^3 \cdot h^{10}$

f. $m^5 \cdot m^2 \cdot m$

g. $(xy)^4$

h. $3d^2 \cdot 7d^5$

i. $(-5a^2b)^3$

Problem 4 Exploring Quotients and Powers

1. Use the definition of a power to write each numerator and denominator as a product.

a. $\frac{9^5}{9^2}$

b. $\frac{5^6}{5^3}$

c. $\frac{z^8}{z^6}$

d. $\frac{u^2}{x}$

2. Simplify each expression you wrote in Question 1. Then, write the simplified expression using exponents.

a.

b.

c.

d.

3. What relationship do you notice between the exponents in the numerator and denominator and the exponents in the simplified expression?

4. Write a rule that you can use to divide with powers.

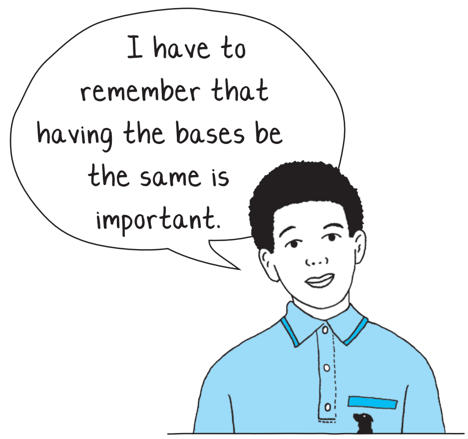
5. Simplify each expression using the rule you wrote for a quotient of powers.

a. $\frac{6^8}{6^3}$

b. $\frac{-9^7}{9^5}$

c. $\frac{2^3}{3^2}$

d. $\frac{12x^4y^5}{2xy^3}$



Problem 5 Who's Correct?

1. Ramon says that $2^6 = 12$. Randy says that $2^6 = 64$. Who is correct? Explain your reasoning.

2. Isabel says that $2^2 + 2^3 = 2^5$, and Elizabeth says that $2^2 + 2^3 \neq 2^5$. Who is correct? Explain your reasoning.

Talk the Talk

In this lesson, you have developed rules for operating with powers. A summary of these rules is shown in the table.

Properties of Powers	Words	Rule
Product Rule of Powers	To multiply powers with the same base, keep the base and add the exponents.	$a^m \cdot a^n = a^{m+n}$
Power to a Power Rule	To simplify a power to a power, keep the base and multiply the exponents.	$(a^m)^n = a^{mn}$
Quotient Rule of Powers	To divide powers with the same base, keep the base and subtract the exponents.	$\frac{a^m}{a^n} = a^{m-n}$, if $a \neq 0$

1. Simplify each expression.

   a. $2a^8 \cdot 2a^6$

   b. $4b^2 \cdot 8b^9$

   c. $-3c \cdot 5c^3 \cdot 2c^9$

   d. $(3d^2)^3$

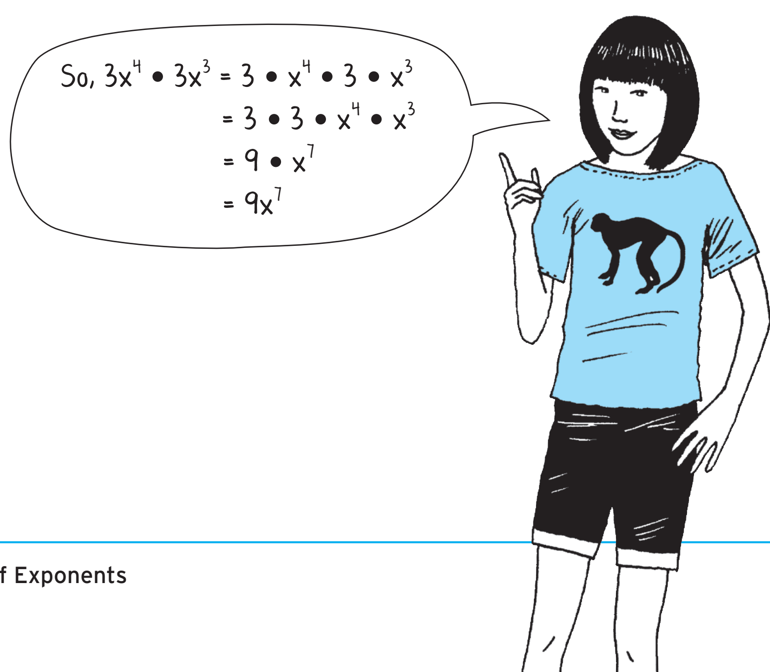
   e. $(10ef^3)^5$

   f. $\frac{f^8}{f^3}$

   g. $\frac{10g^4}{5g^3}$

   h. $\frac{30h^8}{15h^2}$

   i. $\frac{35i^7 j^3}{7i^2 j^3}$



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2. Each expression has been simplified incorrectly. Explain the mistake that occurred, and then make the correction.

a. $(-2x)^3 = 8x^3$

b. $\frac{16x^5}{4x} = 12x^4$

c. $(x^2y^4)^3 = x^6y^7.$

d. $(x^5y^7)(x^2yz) = x^7y^7z$

Be prepared to share your solutions and methods.

7.3 Extending the Rules

Zero and Negative Exponents

Learning Goals

In this lesson, you will:

- Write numbers as powers.
- Simplify powers that have an exponent of zero.
- Simplify powers with negative exponents.

Nada, nil, zilch! These names and many more have been used to describe 0—a number that has always existed, but for centuries was not understood. In fact, the brilliant Greek thinkers struggled with the concept of 0. They couldn't quite grasp that you could have 0 coins or 0 oranges. In other words, they could not make the connection between having "nothing" and giving a numerical value of 0 to "nothing." It wasn't until about 628 AD when the Indian mathematician Brahmagupta established rules for computing with 0.

As you will find out later, dividing a number by 0 is still something of a mystery! Why do you think that dividing a number by 0 is considered "undefined?" What other names for 0 can you think of?

Problem 1 Another Representation for One

1. Simplify each fraction.

   a. $\frac{4}{4}$ b. $\frac{9}{9}$ c. $\frac{25}{25}$

2. What is the quotient of any number divided by itself?

3. Rewrite each numerator and denominator from Question 1 as a power. Do not simplify.

   a. b. c.

4. Simplify each fraction from Question 3 using the Quotient Rule of Powers. Leave your answer as a power.

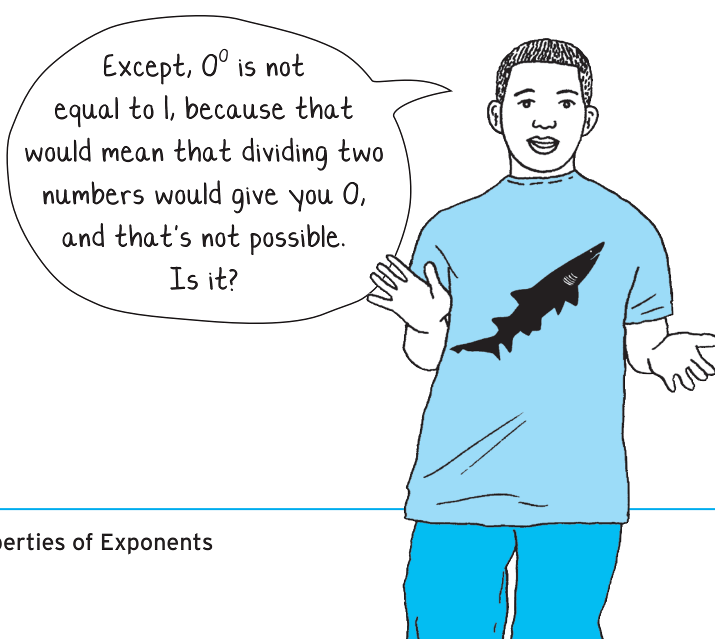
   a. b. c.

5. Although each power in Question 4 has a different base, what is the value of each exponent?

6. Write a rule that you can use when raising any base to the zero power.

7. Use the Quotient Rule of Powers to simplify each expression.

   a. $\frac{6^7}{6^7}$ b. $\frac{4^9}{4^9}$ c. $\frac{12^3}{12^3}$



Problem 2 Exploring Powers that Represent Numbers Less than 1

You know that you can use powers to represent numbers that are greater than 1. Let's determine how to use powers to represent numbers that are less than 1.

1. Let's start with 1 and multiply by 10 three times.

a. Complete the representation. Write each as a power.

	1	=	10^0
Multiply by 10 =	________	=	________
Multiply by 10 =	________	=	________
Multiply by 10 =	________	=	________

b. Describe what happens to the exponents as the number becomes greater.

2. Now, let's start with 1 and divide by 10 three times.

a. Complete the representation. Write the division as a fraction, and then rewrite using the definition of powers. Next, apply the Quotient Rule of Powers, and finally, simplify each expression.

	1	=	$\frac{10^0}{10^0}$	=	10^{0-0}	=	10^0
Divide by 10 =	$\frac{1}{10}$	=	$\frac{10^0}{10^1}$	=	10^{0-1}	=	10^{-1}
Divide by 10 =	________	=	________	=	________	=	________
Divide by 10 =	________	=	________	=	________	=	________

b. Describe what happens to the exponents as the number becomes less.

3. Rewrite each sequence of numbers using the definition of powers.

a. $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8$

b. $\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27$

c. Describe the exponents in the sequence.

4. Simplify each expression using the Quotient Rule of Powers. Show your work.

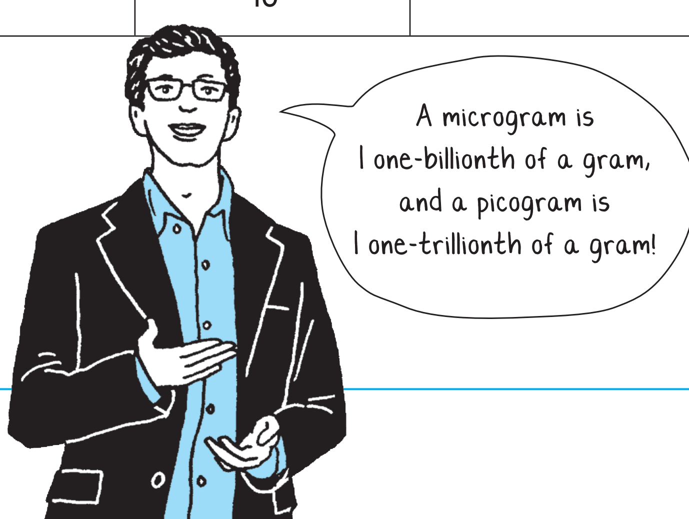
a. $\frac{10^0}{10^3}$

b. $\frac{10^0}{10^5}$

c. $\frac{10^0}{10^4}$

5. Complete the table shown.

Unit	Number of Grams	Number of Grams as an Expression with a Positive Exponent	Number of Grams as an Expression with a Negative Exponent
Milligram	$\frac{1}{1000}$		10^{-3}
Microgram		$\frac{1}{10^6}$	
Nanogram	$\frac{1}{1,000,000,000}$		10^{-9}
Picogram		$\frac{1}{10^{12}}$	



6. Rewrite the power so that the exponent is positive.

a. 8^{-4}

b. 5^{-6}

c. a^{-5}

d. $b^{-2}c^{-3}$

7. Complete the table shown. Use your calculator to determine the value of the expression. (Your answer must be a whole number or fraction. Do not write your answer as a decimal.) Then, rewrite the given expression as an expression with a positive exponent.

Given Expression	Calculator Result (No Decimal)	Expression with a Positive Exponent
$\frac{1}{3^{-2}}$		
$\frac{1}{4^{-2}}$		
$\frac{1}{5^{-2}}$		
$\frac{2^{-2}}{1}$	$\frac{1}{4}$	$\frac{1}{2^2}$
$\frac{3^{-2}}{1}$		
$\frac{5^{-2}}{1}$		

8. Describe how to rewrite any expression that has a 1 in the denominator and a power with a negative exponent in the numerator.

9. Describe how to rewrite any expression that has a 1 in the numerator and a power with a negative exponent in the denominator.

Problem 3 Are They Equal?

The symbol for "is equal to" is =. The symbol for "is not equal to" is ≠. Write the appropriate symbol to compare the expressions in each. Explain your reasoning.

1. $2^3 \; \square \; 2^{-3}$

2. $\frac{1}{2^3} \; \square \; 2^{-3}$

3. $\frac{1}{2^{-3}} \; \square \; 2^3$

4. $\frac{1}{2^{-3}} \; \square \; 2^{-3}$

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Problem 4 Properties of Powers

The expression $(k^5)(k^{-7})$ can be simplified using the Product Rule of Powers or the Quotient Rule of Powers. An expression is not considered simplified if it contains negative exponents.

Using the Product Rule of Powers:

$$(k^5)(k^{-7}) = k^{5+(-7)}$$
$$= k^{-2}$$
$$= \frac{1}{k^2}$$

Using the Quotient Rule of Powers:

$$(k^5)(k^{-7}) = \frac{k^5}{k^7}$$
$$= k^{5-7}$$
$$= k^{-2}$$
$$= \frac{1}{k^2}$$

Simplify each using the properties of powers.

1. $\frac{2^2}{2^6}$

2. $(4x^2)(3x^5)$

3. $(9^4)(9^{-5})$

4. $(8^0)(8^{-2})$

5. $\frac{3^{-3}}{3^{-3}}$

6. $\frac{4^{-2}}{4^{-3}}$

7. $\frac{(-3)^2}{(-3)^4}$

8. $\frac{h^3}{h^5}$

9. $\frac{x^{-4}}{x^5}$

10. $\frac{m^2p^{-2}}{m^4p^3}$

Talk the Talk

A summary of the rules for simplifying powers that you developed in this lesson is shown.

Properties of Powers	Words	Rule	Note
Zero Power	The zero power of any number except for 0 is 1.	$a^0 = 1$, if $a \neq 0$	0^0 is undefined
Negative Exponents in the Numerator	An expression with a negative exponent in the numerator and a 1 in the denominator equals 1 divided by the power with its opposite exponent placed in the denominator.	$a^{-m} = \frac{1}{a^m}$, if $a \neq 0$ and $m > 0$	a^{-m} is read as "a to the opposite of m power." By stating in the rule that $m > 0$, you ensure that the expression a^{-m} has a negative exponent.
Negative Exponents in the Denominator	An expression with a negative exponent in the denominator and a 1 in the numerator equals the power with its opposite exponent.	$\frac{1}{a^{-m}} = a^m$, if $a \neq 0$ and $m > 0$	$\frac{1}{a^{-m}}$ is read as "1 over a to the opposite of m power." By stating in the rule that $m > 0$, you ensure that the expression $\frac{1}{a^{-m}}$ has a negative exponent.

Be prepared to share your solutions and methods.

7.4 Let's Get Scientific!

Scientific Notation

Learning Goals

In this lesson, you will:

- Express numbers in scientific notation.
- Express numbers in standard form.
- Perform operations using scientific notation.

Key Terms

- scientific notation
- order of magnitude
- mantissa
- characteristic

The world is full of interesting and extreme facts. Some of these facts are not always about the greatest or biggest, but sometimes the smallest or least. Here are some interesting facts that you might not know.

- An average pencil can draw a line that is approximately 2,210,000 inches long. That's almost 35 miles!
- The total number of raindrops in an average thundercloud is approximately 1,620,000,000,000,000.
- The thickness of a dollar bill is 0.0043 inch.
- The thickness of a strand of hair is approximately 0.002 inch.

Problem 1 Interesting Facts

Extremely large or small positive numbers can be hard to imagine, or difficult to read. Often, a special mathematical notation is used to make large or small numbers easier to read.

Scientific notation is a notation used to express a very large or a very small number as the product of two numbers:

- A number that is greater than or equal to 1 and less than 10 and
- A power of 10.

Scientific notation makes it much easier to tell at a glance the *order of magnitude*. The **order of magnitude** is an estimate of size expressed as a power of ten. For example, the Earth's mass has an order of magnitude of 10^{24} kilograms.

Use your graphing calculator to explore extremely large and extremely small numbers.

1. Enter each given number into your calculator and complete the table.

Given Number	Calculator Display	What Does the Calculator Display Mean?
35,400,000,000		
60,000,000,000,000		
0.0000007		
0.000008935		

2. Describe the exponents in the powers of 10 for the extremely large numbers.

3. Describe the exponents in the powers of 10 for the extremely small numbers.

4. Describe the number multiplied by the power of 10 in each.

5. Kanye, Corinne, and Brock each tried to write the number 16,000,000,000 in scientific notation. Analyze each student's reasoning. Who is correct?

Brock's Method

I have to write a number greater than 1 and less than 10 multiplied by a power of 10. So, I have to multiply 1.6 by a power of 10. Since there are 9 zeros, my power of 10 will be 10^9. So, 16,000,000,000 is 1.6×10^9.

Corinne's Method

Well, that number is 16 billion. And 16 billion is 16 times 1 billion. 16×1 is the same as 1.6×10, so 16 times 1 billion is the same as 1.6 times 10 billion. I have to multiply 10 ten times to get 10 billion, so my power of 10 is 10^{10}. That means that 16 billion in scientific notation is 1.6×10^{10}.

Kanye's Method

I start with 1.6, a number that is less than 10 and greater than 1. Next, I need a power of 10. If I multiply 1.6 by 10, I get 16. Then, if I multiply by 10 again, I get 160. Multiply by 10 again, and I get 1600. So, I can just keep multiplying by 10 until I get back to the original number. I have to multiply by 10 ten times, so my power of 10 is 10^{10}. So, 16,000,000,000 in scientific notation is 1.6×10^{10}.

In general terms, $a \times 10^n$ is a number written in scientific notation, where a is greater than or equal to 1 and less than 10, and n is any integer. The number a is called the **mantissa,** and n is the called the **characteristic.**

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Problem 2 Getting Scientific in a Big Way!

1. Write each number in either scientific notation or standard notation.

   a. There are approximately 3.34×10^{22} molecules in 1 gram of water. How many molecules are in 1 gram of water?

   b. There are 2.5×10^{13} red blood cells in the human body. How many blood cells are there?

   c. A light year is 5,880,000,000,000 miles. How many miles are there in a light year in scientific notation?

   d. The speed of light is 186,000 miles per second. How fast is the speed of light in scientific notation?

2. The estimated populations, as of July 2009, of several countries are shown. Decide whether the number is written in scientific notation or standard notation. If the number is not in scientific notation, explain how you know it is not. Then, write the number in scientific notation.

   a. People's Republic of China: 1.331×10^{9} people

   b. Pitcairn Islands: 50 people

   c. India: 11.66×10^{8} people

   d. United States: 3.06×10^{8} people

3. List the countries in order of population from least to greatest without writing the numbers in standard notation.

4. Explain how to compare two large numbers that are written in scientific notation.

5. The primary U.S. currency note dispensed at an automated teller machine (ATM) is the 20-dollar bill. There are approximately 6 billion 20-dollar bills in circulation.

 a. Write the approximate number of 20-dollar bills in circulation in standard notation.

 b. Write the number of bills in scientific notation.

 c. Calculate the value of all the 20-dollar bills in circulation.

 d. Write the value you calculated in Question 5, part (c) in scientific notation.

Problem 3 Getting Scientific in a Small Way

Each student tried to write the number 0.00065 in scientific notation. Analyze each student's reasoning.

Brock's Method

I can start with 6.5, which is less than 10 and greater than 1. If I divide by 10, I get 0.65. If I divide by 10 again, I get 0.065. I just keep dividing by 10 until I get to the original number.

0.000[6]5

I divided by 10 four times. So, $0.00065 = \frac{6.5}{10^4}$. But in scientific notation, I have to use multiplication, not division. That's okay because $\frac{6.5}{10^4}$ is the same as $6.5 \times \frac{1}{10^4}$. And since $\frac{1}{10^4}$ is 10^{-4}, I can write 0.00065 in scientific notation as 6.5×10^{-4}.

Corinne's Method

I can write 0.00065 as a fraction less than 1. In words, that decimal is sixty-five hundred thousandths, so I could write it as $\frac{65}{100{,}000}$.

If I divide both the numerator and denominator by 10, I get $\frac{65 \div 10}{100000 \div 10} = \frac{6.5}{10{,}000}$. As a power of 10, the number 10,000 is written as 10^4. So that's $\frac{6.5}{10^4}$, which is the same as $6.5 \times \frac{1}{10^4}$, which is the same as 6.5×10^4. That's the answer.

Kanye's Method

I moved the decimal point in the number to the right until I made a number greater than 1 but less than 10. So, I moved the decimal point four times to make 6.5. And since I moved the decimal point four times to the right, that's the same as multiplying $10 \times 10 \times 10 \times 10$, or 10^4. So, the answer should be 6.5×10^4.

1. Explain what is wrong with Kanye's reasoning.

There are names given to measurements smaller than a meter (m). You are familiar with the centimeter (cm) and the millimeter (mm). Here are some others:

- 1 micrometer (μm) $= \frac{1}{10^6}$ meter
- 1 nanometer (nm) $= \frac{1}{10^9}$ meter
- 1 picometer (pm) $= \frac{1}{10^{12}}$ meter

2. Write each measurement as a power of 10. It is appropriate to have an expression with negative exponents in this question set.

   a. 1 micrometer **b.** 1 nanometer **c.** 1 picometer

3. Write the radius of each type of blood vessel in standard form.

   a. The capillary is one of the minute blood vessels that connect arterioles and venules. The radius of a capillary is 5×10^{-3} mm.

   b. The venule is a small blood vessel that allows deoxygenated blood to return from the capillaries to the veins. The radius of a venule is 1×10^{-2} mm.

   c. The arteriole is a small blood vessel that extends and branches out from an artery and leads to capillaries. The radius of an arteriole is 5.0×10^{-1} mm.

4. Convert each measurement to meters, and then write the measurement in scientific notation.

   a. The diameter of a water molecule is 0.29 nanometers.

   b. The diameter of a red blood cell is 7 micrometers.

   c. The smallest microchip is 0.5 micrometers.

   d. A helium atom has a radius of 31 picometers.

5. Complete the table shown.

Object	Measurement	Measurement in Standard Form	Measurement in Scientific Notation
Earth	Radius in meters		6.38×10^6 m
Brachiosaurus	Mass in kilograms	77,100 kg	
Dust mite	Length in meters	0.00042 m	
Nucleus of an atom	Diameter in meters		1.6×10^{-15} m

Talk the Talk

The table shown lists the types of notation you have seen.

Notation	Definition	Example
Standard	A way in which numbers are normally written.	593
Exponential	A way of writing a number as a power. A power has a base and an exponent.	13^8
Scientific	A way to express a number as a product of a number between 1 and less than 10 and a power of 10	3.87×10^{11}
Decimal	A way to write a number that is less than 1 but greater than 0.	0.64

1. Describe the similarities and differences between the numbers 4.23×10^5 and 4.23×10^{-5}.

7

Be prepared to share your solutions and methods.

7.5 Are We There Yet? What Is the Distance?

Operations with Scientific Notation

Learning Goals

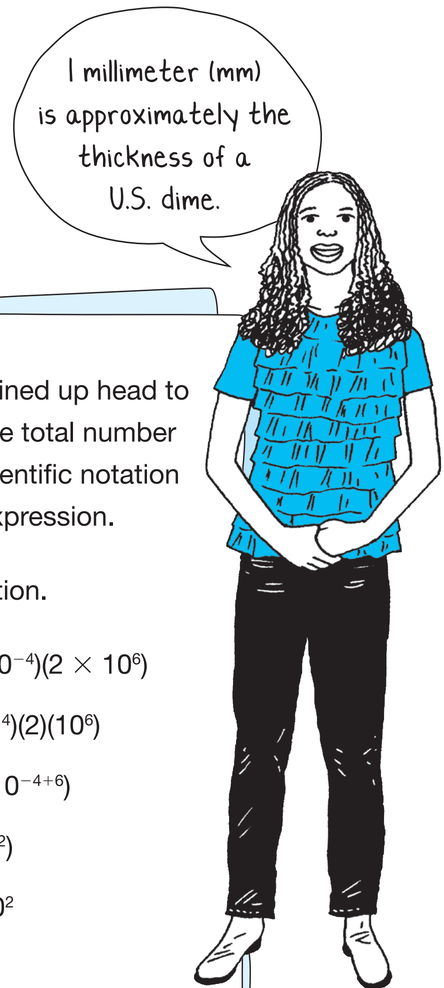
In this lesson, you will:

- Perform operations using scientific notation.
- Perform operations on numbers written in scientific notation.
- Use rules for significant digits in computation.

Truckers never ask, "Are we there yet?" but rather ask, "Where am I going next?" It is estimated that rookie truckers drive about 2500 miles per week. If a rookie trucker were to drive every week of the year, how many miles would they drive in 1 year? How many miles would a trucker drive in 20 years? How would you write this number in scientific notation?

Problem 1 Applying the Product Rules of Powers to Scientific Notation

There are many very small insects in the world. Some beetles are less than one millimeter in length. Fairyflies are tiny wasps that are one of the smallest insects known; they are approximately 0.0002 meter in length. If 2,000,000 fairyflies were lined up head to tail, how far would they stretch?



To calculate the total length of 2,000,000 fairyflies lined up head to tail, you would multiply the length of 1 fairyfly by the total number of fairyflies, or (0.0002)(2,000,000). You can use scientific notation and the Product Rules of Powers to simplify this expression.

Begin by converting the numbers to scientific notation.

	$(2 \times 10^{-4})(2 \times 10^{6})$
Think about the product as:	$(2)(10^{-4})(2)(10^{6})$
Apply the Product Rule of Powers:	$(2^{1+1})(10^{-4+6})$
Simplify each power:	$(2^{2})(10^{2})$
Rewrite in scientific notation:	4×10^{2}
Simplified:	400

If you lined up 2,000,000 fairyflies head to tail, they would span 400 meters.

1. An ecologist estimates that it takes approximately 196,000 pounds of buried plant matter to produce one gallon of gasoline. Some energy experts estimate that the United States consumed about 140 billion gallons of gasoline in 2008.

   Calculate the amount of buried plant matter needed to produce the amount of gasoline consumed in 2008. Write your answer in scientific notation. Explain your reasoning.

2. An oil tanker is approximately 1400 feet long. How far would 9500 oil tankers span if they were placed end to end?

   a. Calculate the approximate length of 9500 oil tankers. Write your answer in scientific notation. Explain your reasoning.

   b. What additional step is required to calculate the answer in Question 2, part (a)?

3. Calculate each product. Express each product in scientific notation.

   a. $(3 \times 10^5)(2 \times 10^6)$

   b. $(9 \times 10^4)(1 \times 10^7)$

   c. $(4.0 \times 10^8)(2.7 \times 10^4)$

   d. $(5.6 \times 10^{-6})(3.5 \times 10^{15})$

4. Matthew and Lorraine are checking answers to their homework. Matthew shares 73.5×10^{16} as the answer to a planet distance problem. Lorraine says that the answer was 7.35×10^{17}. Who is correct? Explain your reasoning.

5. What is the missing factor in each expression? Explain your reasoning.

a. $(4 \times 10^7)(? \times ?) = 8 \times 10^{12}$

b. $(? \times ?)(5 \times 10^3) = 3.5 \times 10^8$

Problem 2 Applying the Quotient Rule of Powers to Scientific Notation

The Scoville scale measures the hotness of a chili pepper by the amount of capsaicin it contains. Capsaicin is the chemical that puts the "heat" in chili peppers. The number of Scoville heat units (SHU) indicates the amount of capsaicin present in the food. The table represents the Scoville rating for a variety of peppers.

Scoville Rating	Type
15,000,000 to 16,000,000	Pure Capsaicin
5,000,000 to 5,300,000	Law Enforcement Grade Pepper Spray
350,000 to 570,000	Red Savina Habanero
200,000 to 300,000	Habanero
70,000 to 80,000	Thai Pepper
30,000 to 50,000	Cayenne Pepper, Tabasco Pepper, some Chipotle Peppers
2500 to 8000	Jalapeño Peppers, Paprika (Hungarian)
500 to 2500	Anaheim Pepper (Mild Chile Pepper)
100 to 500	Pimento, Pepperoncini
0	No Heat, Bell Pepper

*Source: Mojave Pepper Farm's Pepper Scale

Use the values from the table to answer each question.

1. How many times hotter is the mildest law enforcement pepper spray than the hottest pepperoncini?
   a. First, write a ratio using the values in the table as they appear.
   b. Next, write your ratio in scientific notation.
   c. Finally, simplify your expression using the Quotient Rule of Powers. Explain your reasoning.
   d. What does your simplified result represent?

2. How many times hotter is the hottest red savina habanero than the mildest jalapeño pepper?
   a. First, write a ratio using the values in the table as they appear.
   b. Next, write your ratio in scientific notation.
   c. Finally, simplify your expression using the Quotient Rule of Powers. Explain your reasoning.
   d. What does your simplified result represent?

3. Simplify each expression. Express the quotient in scientific notation.

a. $\frac{(6 \times 10^8)}{(2 \times 10^3)}$

b. $\frac{(9 \times 10^5)}{(3 \times 10^9)}$

c. $\frac{(8 \times 10^{-4})}{(2 \times 10^3)}$

Problem 3 Adding and Subtracting Numbers in Scientific Notation

The table shows the average distance of each planet from the Sun.

Planet	Average Distance from the Sun (miles)	Average Distance from the Sun Written in Scientific Notation (miles)
Mercury	58,000,000	
Venus	108,000,000	
Earth	149,600,000	
Mars	228,000,000	
Jupiter	778,500,000	
Saturn	1,430,000,000	
Uranus	2,880,000,000	
Neptune	4,500,000,000	

1. Write each distance in scientific notation in the table.

2. On average, how much farther is Mars than Venus from the Sun?

3. Write the answer from Question 2 in scientific notation.

4. Look at the exponents for the distance from the Sun to Mars, Sun to Venus, and the exponent of your answer from Question 3. What do you notice?

5. Look at the mantissa for the distance from the Sun to Mars, Sun to Venus, and the mantissa of your answer from Question 3. What do you notice?

6. Describe how to calculate the difference of two numbers written in scientific notation that have the same exponent.

7. On average, how much farther is Mars than Mercury from the Sun?

8. Write the answer from Question 7 in scientific notation.

9. How is the answer you calculated in Question 8 different from the answer you calculated in Question 3?

Carlos and Tonya are given the problem $1.2 \times 10^3 + 5.3 \times 10^5$. Each solves it correctly, but in different ways.

Carlos

First, I rewrote 1.2×10^3 as 0.012×10^5. Then I added.

$$1.2 \times 10^3 + 5.3 \times 10^5 = 0.012 \times 10^5 + 5.3 \times 10^5$$
$$= 5.312 \times 10^5$$

Tonya

I rewrote 5.3×10^5 as 530×10^3, and then I added.

$$1.2 \times 10^3 + 5.3 \times 10^5 = 1.2 \times 10^3 + 530 \times 10^3$$
$$= 531.2 \times 10^3$$
$$= 5.312 \times 10^5$$

10. How are their methods similar? How are their methods different?

11. Describe how to calculate the sum of two numbers written in scientific notation that have different exponents.

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The sum $1.2 \times 10^3 + 5.3 \times 10^5$ can also be calculated using common factors.

$(1.2 \times 10^3) + (5.3 \times 10^5)$	
$10^3 [(1.2) + (5.3 \times 10^2)]$	Factor out the greatest common factor.
$10^3(1.2 + 530)$	Rewrite (5.3×10^2).
$10^3(531.2)$	Combine like terms.
$(531.2)10^3$	Use the Commutative Property of Multiplication.
531.2×10^3	
5.312×10^5	Write in proper scientific notation.

12. Calculate each sum or difference.

a. $3.7 \times 10^5 + 2.1 \times 10^6$

b. $2.9 \times 10^8 - 1.4 \times 10^4$

c. $2.5 \times 10^4 - 3.1 \times 10^2$

d. $9.1 \times 10^8 + 4.3 \times 10^7$

Problem 4 Powers of Numbers in Scientific Notation

1. Previously, you used the properties of exponents to simplify expressions such as $(-2xy^3)^2$.

   Describe each step.

   $(-2xy^3)^2$

   $(-2)^2(x)^2(y^3)^2$ ______________________________

   $4x^2y^6$ ______________________________

2. Calculate $(4 \times 10^3)^2$ using a similar procedure. Write your answer in scientific notation.

3. Calculate each power. Write your answer in scientific notation.

   a. $(3 \times 10^2)^4$

   b. $(1.7 \times 10^5)^3$

   c. $(4.5 \times 10^3)^3$

   d. $(1.2 \times 10^4)^6$

Be prepared to share your solutions and methods.

7.6 Watch Your Step!

Identifying the Properties of Powers

Learning Goals

In this lesson, you will:

- Review the Power of a Power Property.
- Review the Power of a Product Property.
- Review the Power of a Quotient Property.
- Review multiplication and division of numbers written in scientific notation.

So, you have been performing calculations with powers, but do you know where they actually came from? Well, it appears that the word "exponent" was first used in a book entitled **Arithmetica Integra** written by English mathematician Michael Stifel in 1544. However, the concept of exponents may have actually come from the Babylonians, who used a number system that was quite different from the number system we use today. The Babylonians had tables of square numbers and cube numbers nearly 5000 years ago!

Do you think that there are still mathematical discoveries that can occur? Or do you think all mathematical concepts have been discovered.

Problem 1 Show What You Know

1. Create graphic organizers for each.

   Product Rule of Powers

   Quotient Rule of Powers

   Power of a Power Rule

   Zero Power Rule

   Negative Exponent Rule

   Scientific Notation

As you are creating your representations, consider the following:

Definition in your own words:	How would you describe this property to a friend?
Facts/Characteristics:	Does this property work the same for variables and numbers? Are there specific characteristics if the numbers are positive or negative?
Examples:	Include examples with variables and different types of numbers (e.g., positive, negative, and fractions).
General Rule:	Use variables. Be mindful when your variable cannot be zero.

Definition in Your Own Words

Facts/Characteristics

Product Rule of Powers

Examples

General Rule

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DEFINITION IN YOUR OWN WORDS

FACTS/CHARACTERISTICS

QUOTIENT RULE OF POWERS

EXAMPLES

GENERAL RULE

DEFINITION IN YOUR OWN WORDS

FACTS/CHARACTERISTICS

POWER OF A POWER RULE

EXAMPLES

GENERAL RULE

DEFINITION IN YOUR OWN WORDS

FACTS/CHARACTERISTICS

ZERO POWER RULE

EXAMPLES

GENERAL RULE

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DEFINITION IN YOUR OWN WORDS

FACTS/CHARACTERISTICS

NEGATIVE EXPONENT RULE

EXAMPLES

GENERAL RULE

DEFINITION IN YOUR OWN WORDS

FACTS/CHARACTERISTICS

SCIENTIFIC NOTATION

EXAMPLES

GENERAL RULE

Problem 2 Justifying the Steps

1. Identify the rule that justifies each step to simplify the expression. Use the example shown.

$$\left(\frac{2x^5}{y^4}\right)^3 =$$

$$= \frac{(2x^5)^3}{(y^4)^3}$$ Power to a Power Rule

$$= \frac{2^3x^{15}}{y^{12}}$$ Power to a Power Rule

$$= \frac{8x^{15}}{y^{12}}$$ Simplify the Powers

a. $(2a^3c^2)^4(-4ac^4)^3 =$

$= (2^4a^{3(4)}c^{2(4)})((-4)^3a^3c^{4(3)})$ ____________________

$= (16a^{12}c^8)(-64a^3c^{12})$ ____________________

$= (16)(-64)(a^{12}a^3)(c^8c^{12})$ ____________________

$= -1024a^{15}c^{20}$ ____________________

b. $\frac{(3d^2e^2)^3}{(2d^4e^3)^2} =$

$= \frac{3^3d^{2(3)}e^{2(3)}}{2^2d^{4(2)}e^{3(2)}}$ ____________________

$= \frac{27d^6e^6}{4d^8e^6}$ ____________________

$= \frac{27e^{(6-6)}}{4d^{(8-6)}}$ ____________________

$= \frac{27e^0}{4d^2}$ ____________________

$= \frac{27}{4d^2}$ ____________________

2. Simplify each expression using the properties of powers. Express your answers using only positive exponents.

a. $\dfrac{(2r^3s^5)}{(4r^3s^2)}$

b. $\dfrac{(4j^2k^3)^4}{(2j^3k^2)^3}$

c. $\dfrac{(3n^2m^4)^8}{(3n^2m^4)^8}$

d. $\dfrac{(10^5 \cdot 10^5)}{10^6}$

e. $\left(\dfrac{2x^3}{y}\right)^3 \cdot \left(\dfrac{1}{6x^3}\right)$

Problem 3 Who's Correct?

Determine which student(s) used the properties of powers correctly. Explain why the other expressions are not correct.

1. $\frac{g^7h^4}{g^3h^9}$

   Adam wrote $g^{10}h^{13}$.

   Nic wrote $\frac{g^4}{h^5}$.

   Shane wrote g^4h^5.

Who is correct?

2. $\frac{2w^{-4}}{x^{-2}}$

   Adam wrote $\frac{2x^2}{w^4}$.

   Nic wrote $\frac{x^2}{2w^4}$.

   Shane wrote $\frac{2w^4}{x^2}$.

Who is correct?

Be prepared to share your solutions and methods.

7

Chapter 7 Summary

Key Terms

- power (7.1)
- base (7.1)
- exponent (7.1)
- scientific notation (7.4)
- order of magnitude (7.4)
- mantissa (7.4)
- characteristic (7.4)

Rules

- Product Rule of Powers (7.6)
- Quotient Rule of Powers (7.6)
- Power to a Power Rule (7.6)
- Zero Power Rule (7.6)
- Negative Exponent Rule (7.6)

7.1 Writing a Power as a Product

A power is made up of a base and an exponent. An exponent tells you how many times to multiply a base by itself.

Example

Write the power 3^5 as a product. Then calculate the product.

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3$$
$$= 243$$

7.2 Writing a Product as a Power

A power is a shorter way to write a product. To write a product as a power, the same factor must be multiplied repeatedly. This repeated factor is the base.

Example

Write $2 \times 2 \times 2 \times 2 \times 2 \times b \times b \times c \times c \times c \times c$ as a product of powers.

$$2 \times 2 \times 2 \times 2 \times 2 \times b \times b \times c \times c \times c \times c = 2^5b^2c^4$$

7.2 Using Properties of Exponents to Operate with Powers

To multiply powers with the same base, keep the base and add the exponents. To raise a power to a power, keep the base and multiply the exponents. To divide powers with the same base, keep the base and subtract the exponents.

Example

Simplify the expression $\frac{p(p^2q^3)^3}{q^2}$.

$$\frac{p(p^2q^3)^3}{q^2} = \frac{p(p^{2(3)}q^{3(3)})}{q^2}$$

$$= \frac{p(p^6q^9)}{q^2}$$

$$= \frac{p^{1+6}q^9}{q^2}$$

$$= \frac{p^7q^9}{q^2}$$

$$= p^7q^{9-2}$$

$$= p^7q^7$$

7.3 Simplifying Expressions with Zero and Negative Exponents

Any number except for 0 raised to the zero power is 1. A negative exponent in the numerator of a fraction can be rewritten as a positive exponent in the denominator of the fraction. Similarly, a negative exponent in the denominator of a fraction can be rewritten as a positive exponent in the numerator of the fraction.

Example

Simplify the expression $\frac{x^0y^{-5}}{z^{-4}}$.

$$\frac{x^0y^{-5}}{z^{-4}} = \frac{1(y^{-5})}{z^{-4}}$$

$$= \frac{z^4}{y^5}$$

7

7.4 Expressing Large Numbers in Scientific Notation

Scientific notation is a notation used to express a very large or very small number as the product of two numbers:

- A number greater than or equal to 1 and less than 10, and
- A power of 10.

Large numbers written in scientific notation will have a positive exponent as the characteristic.

Example

A classmate incorrectly writes the number 875,000,000,000 in scientific notation as 875×10^9. Explain why your classmate is wrong and make the correction.

Scientific notation requires the mantissa to be a number greater than or equal to 1 and less than 10. The correct answer is 8.75×10^{11}.

7.4 Expressing Small Numbers in Scientific Notation

Small numbers written in scientific notation will have a negative exponent as the characteristic.

Example

Write 0.00000036 in scientific notation.

$0.00000036 = 3.6 \times 10^{-7}$

7.5 Performing Operations on Numbers Written in Scientific Notation

The properties of exponents apply to numbers expressed in scientific notation.

Example

Simplify $\frac{(2.6 \times 10^5)(4.5 \times 10^3)}{3.1 \times 10^4}$.

$$\frac{(2.6 \times 10^5)(4.5 \times 10^3)}{3.1 \times 10^4} = \frac{11.7 \times 10^{5+3}}{3.1 \times 10^4}$$

$$= \frac{11.7 \times 10^8}{3.1 \times 10^4}$$

$$\approx 3.8 \times 10^{8-4}$$

$$\approx 3.8 \times 10^4$$

7.6 Identifying the Properties of Powers

The properties of powers can be used to justify each step taken to simplify an expression. These properties include the Product Rule of Powers, Quotient Rule of Powers, Power to a Power Rule, Zero Power Rule, and Negative Exponent Rule.

Example

Identify the property that justifies each step to simplify the expression.

$\frac{3x^9(6x^2y^5)}{(x^2y)^5z^{-1}} = \frac{18x^{11}y^5}{(x^2y)^5z^{-1}}$	Product Rule of Powers
$= \frac{18x^{11}y^5}{x^{10}y^5z^{-1}}$	Power to a Power Rule
$= \frac{18x^{11}y^5z}{x^{10}y^5}$	Negative Exponent Rule
$= 18xy^0z$	Quotient Rule of Powers
$= 18xz$	Zero Power Rule