12 MULTIPLE REPRESENTATIONS OF LINEAR FUNCTIONS

You probably see people jogging or running in your neighborhood all the time. Jogging regularly helps keep your heart and lungs healthy, helps to strengthen your immune system, reduces stress, and can even improve your mood!

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12.1 WILLIE CATCHEM

Analyzing Problem Situations Using Multiple Representations

Learning Goal

In this lesson, you will:

▶ Analyze problem situations using multiple representations.

A triathlon is a race that involves three different challenges—swimming, cycling, and running. In the Ironman triathlon, competitors must swim for 3.8 kilometers, cycle for 180 kilometers, and run for 42.2 kilometers.

All together, the Ironman triathlon is over 140 miles long! That's about the same distance to drive from Portland, Oregon, to Tacoma, Washington. Can you estimate a location on a map that is about 140 miles from where you live? How long do you think it would take you to swim, ride, and run from your house to that location?

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Problem 1 Through the Origin

Willie Catchem is a long-distance runner. His training includes running about 240 kilometers per week. Willie's training route goes by Kevin's house at the beginning of his run. Willie also runs past Kevin's house near the end of his run.

One day Kevin wanted to determine how fast Willie actually ran. Kevin knows that Willie's complete running route is 12 kilometers.

The table shows Willie's times.

Time (min)	Distance (km)
0	0
40	12

1. Calculate Willie's average speed in kilometers per minute.

   Average speed =

2. Define variables for the time and distance Willie runs.

3. Write an equation that shows the relationship between these variables.

4. If Willie ran for 15 minutes and he maintains his average speed, how far would he run?

5. Willie often runs 10K (10-kilometer) races. How long would it take for Willie to finish a 10K race if he runs at his average speed?
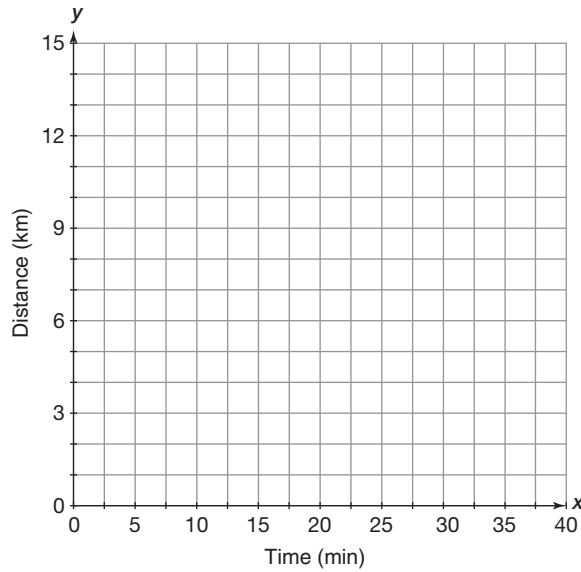
There are about 6.2 miles in a 10K race.
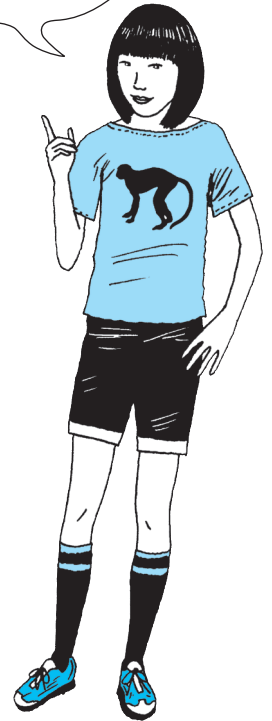
6. Complete the table using the information from Questions 1 through 5.

Time (min)	Distance from Kevin's House (km)
0	
15	
	10
	12

12

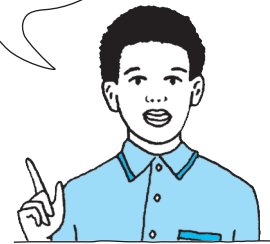**7.** Graph the points on the coordinate plane shown.

8. Does it make sense to connect the points for this problem situation? Why or why not?

9. Describe the type of function represented.

To determine the type of function consider the shape of the graph and where it crosses the y-axis.

10. Which is the independent variable? Which is the dependent variable?

11. What is the slope? What is the *y*-intercept? Explain the meaning of each in terms of the problem situation.

12. If it takes Willie 18 minutes to get to Kevin's house from his own house, how far away is Willie's house?

Problem 2   A Different Intercept

Lisa runs the same route as Willie, but her house is in between Willie's and Kevin's houses. Lisa uses her watch to clock her running time, using Kevin's house as her marker. The table shows the watch readings each time she passes Kevin's house.

Time (min)	Distance from Kevin's House (km)
8.5	0
47.25	12

> The first time shown is not 0 minutes. How will that change the problem?

1. Calculate Lisa's average speed in kilometers per minute.

Average speed =

2. Assume that Lisa runs at this same speed. How far from Kevin's house is Lisa's house?

3. Define variables for the time it takes Lisa to run from her house, and her distance from Kevin's house.

4. Write an equation that shows the relationship between the variables.

5. If Lisa started at her house and ran for 10 minutes and maintains her average speed during her run, how far would she be from Kevin's house?

6. If Lisa is 6 kilometers past Kevin's house, how long has she been running since she left home?

7. Use the points and the information you have about Lisa's time running to complete the table.

Time from Her House (min)	Distance from Kevin's House (km)
0	
10	
	6

8. Graph the points on the coordinate plane shown.
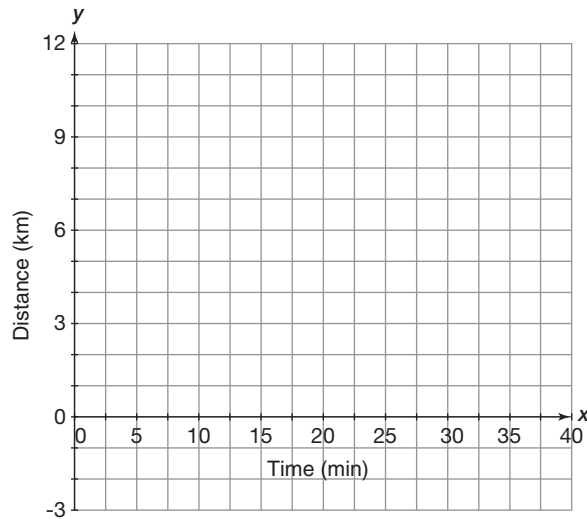
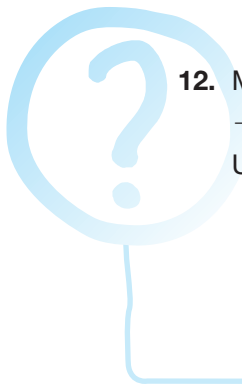**9.** Does it make sense to connect the points in this problem situation? Why or why not?

10. Describe the type of function represented.

How does the function representing Lisa's average speed compare to the function representing Willie's average speed?

11. Which is the independent variable? Which is the dependent variable?

12. Marty claims that the slope for the problem situation is −2.635 and the *y*-intercept is 0.31. Samantha doesn't think that Marty is correct. Using only the graph you created, who is correct? Explain your reasoning.

13. What is the slope? What is the *y*-intercept? Explain the meaning of each in terms of the problem situation.

14. What does the *y*-intercept mean in terms of this problem situation?

15. Who is the faster runner, Lisa or Willie? How do you know?

How did each representation, the scenario, table, graph, and equation, help you determine who was the faster runner?

Be prepared to share your solutions and methods.

12.2

PONY EXPRESS
Interpreting the Standard Form of a Linear Equation

Learning Goals

In this lesson, you will:

▶ Use the standard form of a linear equation to represent a problem situation.

▶ Use the standard form to analyze and solve problems.

▶ Identify the meaning and value of the component expressions in the standard form of a linear equation.

In 1860, the Pony Express began as a way to deliver the news and mail faster. It consisted of relays of men riding horses carrying saddlebags of mail across a 2000-mile trail. The service started on April 3, 1860, when riders left simultaneously from St. Joseph, Missouri, and Sacramento, California. The first westbound trip was made in 9 days and 23 hours, and the eastbound journey in 11 days and 12 hours. Eventually, the Pony Express had more than 100 stations, 80 riders, and between 400 and 500 horses.

The express route was very dangerous, but only one mail delivery was ever lost. The service lasted only until the completion of the Pacific Telegraph line, which made communicating much quicker than mail. Although California relied upon news from the Pony Express during the early days of the Civil War, the horse line never made much money. In fact, the founders of the Pony Express had to file for bankruptcy. However, the romantic drama surrounding the Pony Express has made it a part of the legend of the American West. Can you think of other ways people also communicated before the U.S. Postal Service or email?
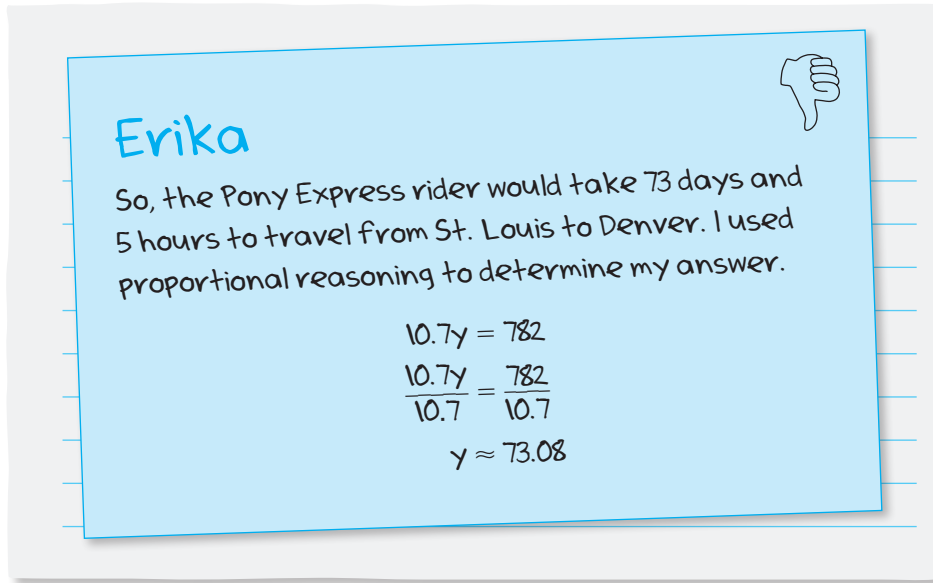
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Problem 1   The Stagecoach and the Pony Express

In the mid-1800s, delivering mail and news across the American Great Plains was time-consuming and made for a long delay in getting vital information from one side of the country to the other. At the time, most mail and news traveled by stagecoach along the main stagecoach lines at about 8 miles per hour. The long stretch of 782 miles from the two largest cities on either side of the plains, St. Louis, Missouri, and Denver, Colorado, was a very important part of this trail.

1. Use the variable x for the time that a stagecoach was driven in hours, and write an expression to represent the distance the stagecoach was driven in miles.

2. How many miles would the stagecoach travel in:

   a. 8 hours?

   b. 10 hours 30 minutes?

3. In how many hours would the stagecoach travel:

   a. 200 miles?

   b. 150 miles?

4. How long would it take the stagecoach to travel from St. Louis to Denver?

5. The Pony Express riders averaged about 10.7 miles per hour. Use the variable y for the time that the Pony Express rider rides in hours. Then, write an expression to represent the distance that he rides in miles.

6. How many miles would the rider travel in:

    a. 7 hours?

    b. 11 hours 15 minutes?

7. In how many hours would the rider travel:

    a. 100 miles?

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b. 600 miles? (hours and minutes)

8.

Erika

So, the Pony Express rider would take 73 days and 5 hours to travel from St. Louis to Denver. I used proportional reasoning to determine my answer.

$$10.7y = 782$$
$$\frac{10.7y}{10.7} = \frac{782}{10.7}$$
$$y \approx 73.08$$

Explain to Erika why her calculation is incorrect and determine the correct answer.

Problem 2   What If Both Were Used?

Sometimes the mail may have been delivered using a stagecoach for part of the route and the Pony Express for the remainder of the route.

1. Write an expression for the distance that was traveled using both of these delivery methods on one trip.

2. Write an equation that represents using this method to deliver the mail from St. Louis to Denver.

3. If the Pony Express rode for 20 hours from St. Louis before handing off the mail to a stagecoach, how long would it take the stagecoach to get the mail to Denver?
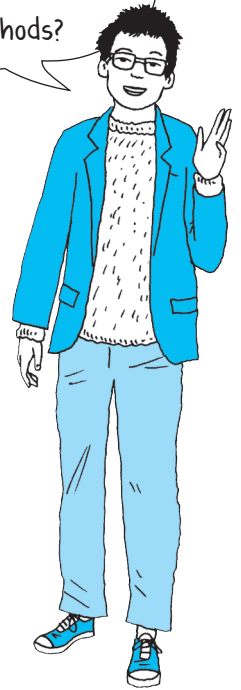
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4. If the stagecoach traveled for 50 hours from St. Louis before handing off the mail to a Pony Express rider, how long would it take the rider to get the mail to Denver?

Do your answers make sense? Did you check your work?

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5. Complete the table.

Delivery Time of Mail Between St. Louis and Denver

Time the Mail Was in a Stagecoach (hours)	Time the Mail Was with the Pony Express (hours)
	20
50	
	50
8	
	7
	56.07

6. Graph the points from the table on the coordinate plane and label the axes.

Delivery Time of Mail Between St. Louis and Denver
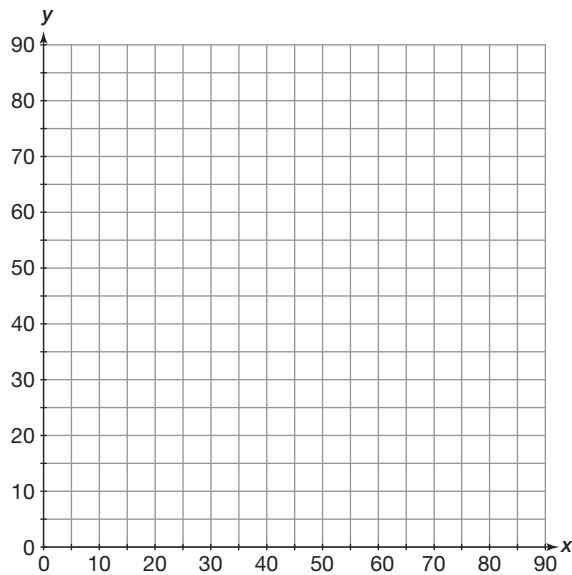
7. Describe the points on the graph.

8. Does it make sense to connect the points in this problem situation? Why or why not?

9. Based on this graph, describe the type of function represented by this equation.

10. Analyze the table and graph you completed.

    a. As the *x*-value increases, what happens to the *y*-value?

    b. As the *y*-value increases, what happens to the *x*-value?

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Problem 3 The Parts

1. If the Pony Express took the mail for 400 miles, how many miles would the stagecoach need to travel to complete the trip from St. Louis to Denver?

2. How much time would it take for the stagecoach to finish this trip?

3. If the stagecoach took the mail for 300 miles, how many miles would the Pony Express rider need to complete the trip from St. Louis to Denver?

4. How much time would it take for the rider to finish this trip?

Your equation $8x + 10.7y = 782$ is in the standard form of a linear equation, $ax + by = c$, where a, b, and c are constants.

5. What are the values of a, b, and c in the equation you wrote for mail deliveries between stagecoaches and the Pony Express?

6. Describe what each constant, variable, or other expression represents in this equation.

   a. *x*

   b. *y*

   c. 8*x*

   d. 10.7*y*

   e. 8*x* + 10.7*y*

   f. 782

7. Describe the meaning of each in this problem situation.

   a. *x*-intercept

   b. *y*-intercept

Be prepared to share your solutions and methods.

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12.3 SLOPES, FORMS, GRAPHS, AND INTERCEPTS

Connecting the Standard Form with the Slope-Intercept Form of Linear Functions

Learning Goals

In this lesson, you will:

▶ Graph linear functions in standard form.

▶ Transform linear functions from one form to the other.

▶ Determine the slope and the intercepts of linear equations in standard form.

Many different kinds of animals can change their form to help them avoid or ward off predators. Some chameleons can change color to camouflage themselves so that predators don't see them. When threatened, a frill-necked lizard can raise up and expand the skin around its neck to make it look more dangerous.

But the vampire squid has one of the most unusual ways of changing form when it feels threatened. A vampire squid will literally turn itself inside out!

You have done a lot of work with changing form in mathematics—but hopefully not because you were threatened! What ways have you changed from one form to another in math?

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Problem 1 Exploring Linear Equations in Standard Form

Complete the table for each linear equation shown in standard form. Use the *x*- or *y*-value given to determine the unknown value. Use the third column to show your work.

1. $4x + 3y = 12$

x	y	Work
2		
	5	
−2		

2. $5x - 4y = 20$

x	y	Work
6		
	−4	
0		

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3. $3x - 5y = 30$

x	y	Work
0		
	0	
−3		

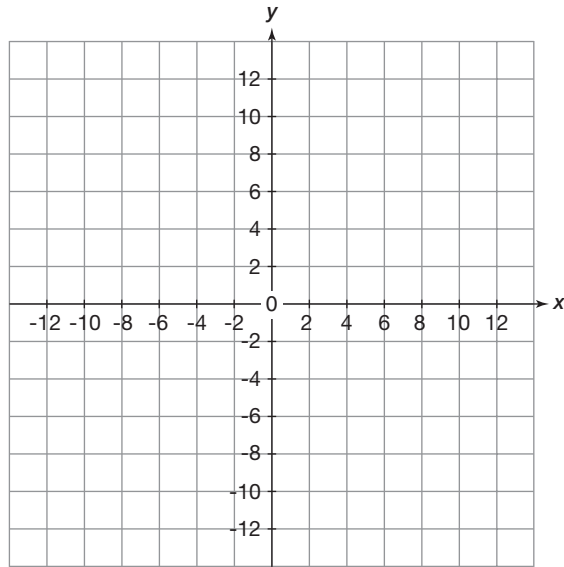**4.** Plot the points from the tables to graph each equation on the coordinate plane.

Label each line with its equation.

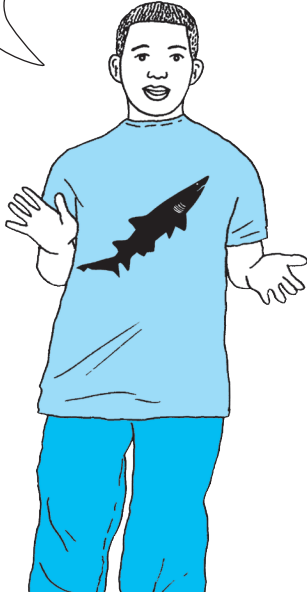**5.** Use two points from each table to calculate the slope for each equation.

a. $4x + 3y = 12$

$m =$

Check out your graphs to get a sense of each slope before you use the slope formula. Which graphs will have a positive slope? Which graphs will have a negative slope?

b. $5x - 4y = 20$

$m =$

c. $3x - 5y = 30$

$m =$

6. Complete the table by using the equation in standard form and the given value to determine the unknown x- and y-intercepts and a third point. Then, graph the equation on the coordinate plane provided.
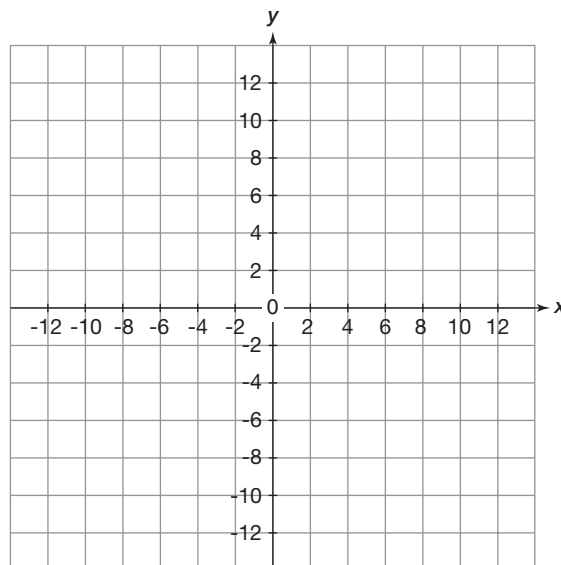
a. $2.5x + 5y = 15$

x	y	Work
0		
	0	
−1		

x-intercept:

y-intercept:

Slope: $m =$

b. $\frac{2}{3}x - 6y = 12$

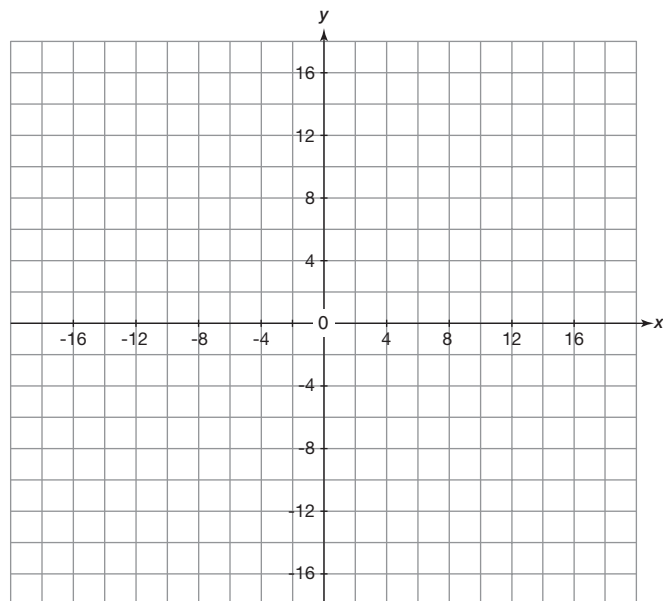| x	y	Work
0		
	0	

x-intercept:

y-intercept:

Slope: $m =$

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7. Using the information from Question 6, rewrite each linear function in slope-intercept form.

Standard Form	Slope-Intercept Form
$2.5x + 5y = 15$	
$\frac{2}{3}x - 6y = 12$	

> I prefer standard form when I am looking for the x- and y-intercepts.

> I prefer slope-intercept form. I can tell what the slope is and what the y-intercept is just by looking at the equation.

Problem 2   Transform from One to the Other

To convert a linear function in standard form to slope-intercept form, solve for y.

1. Rewrite each equation in standard form to slope-intercept form.

a. $4x + 8y = 12$

b. $2.5x - 3.5y = 7$

c. $\frac{3}{4}x + \frac{1}{3}y = 7$

d. $11x - 25y = 185$

e. $ax - by = c$

This equation shows how you can determine the slope and y-intercept even in a standard form equation.

To convert a linear equation in slope-intercept form to standard form, solve for the constant term.

2. Rewrite each equation in standard form.

a. $y = 3x - 7$
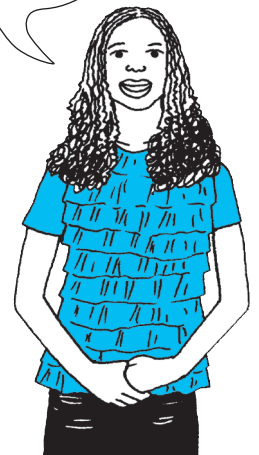
b. $y = \frac{3}{4}x - \frac{7}{5}$

c. $y = -3.5x + 7.8$

Now I see how to write any slope-intercept form equation in standard form.

d. $y = mx + b$

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Talk the Talk

Describe how to determine the slope, *y*-intercept, and *x*-intercept from each form of an equation.

1. $y = mx + b$

slope:

y-intercept:

x-intercept:

2. $ax + by = c$

slope:

y-intercept:

x-intercept:

Be prepared to share your solutions and methods.

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12.4 THE JOURNEY STARTS WITH A SINGLE STEP—BUT THERE ARE MANY STEPS AFTER THAT!

Intervals of Increase, Decrease, and No Change

Learning Goals

In this lesson, you will:

▶ Analyze a problem situation using multiple representations.

▶ Identify intervals of increase, decrease, and constant values of a function.

Key Terms

▶ increasing function
▶ constant function
▶ decreasing function
▶ interval of increase
▶ interval of decrease
▶ constant interval

Overland travel is a long journey often traveling between countries. Overland traveling can involve traveling by train, bus, bike, or boat, but never by plane. People often travel with a group for weeks or months to get to their destinations. Some of the longest overland travel routes are in Africa. The Cairo to Cape Town route covers more than 10,000 kilometers and follows the Nile River through a number of African countries. This journey can take up to 4 months to complete! Some people have even overland traveled from London to South Africa. How long do you think that journey would take? Would you want to go on an overland travel journey?

Problem 1   Heading to the Basketball Courts

James is visiting at Tyler's house about 2000 feet from home. He then decides to head to the park to play basketball. From Tyler's house, James walks at the rate of 8 feet per second and it takes him 25 minutes to arrive at the basketball courts.

| James's House | Tyler's House | Basketball Courts |

Liberty Avenue

N
W ⊥ E
S

1. How far away from home is James when he arrives at the basketball courts?

> About how many miles is 2000 feet?

2. Define variables for the time that James walks in seconds, and his distance in feet from home.

3. Write an equation that relates these variables.

4. What is the slope and the *y*-intercept of this function? Explain what each represents in this problem situation.

5. Complete the table to select appropriate upper and lower bounds to graph this function.
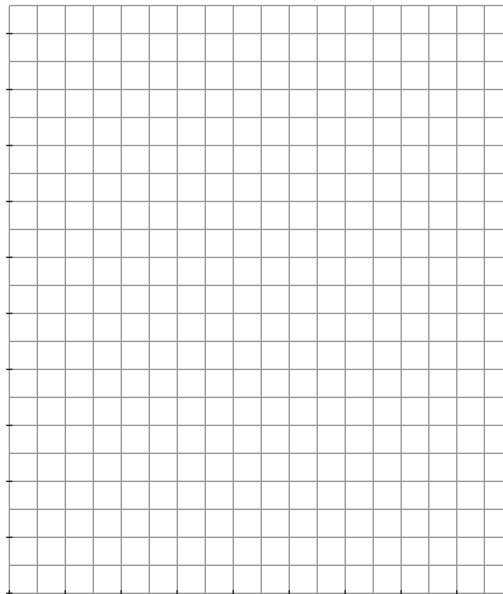
Quantity	Lower Bound	Upper Bound
Time		
Distance		

Remember, when setting the lower and upper bounds think about how to surround the data.

6. What is the domain of this problem situation? What is the range?

7. Graph the function and label each axis.

8. As the time increases, what happens to the distance?

When both values of a function increase, the function is said to be an **increasing function.**

Problem 2   Shooting Hoops

James arrives at the park and has one hour to play basketball.

1. How far is James from home?

2. After being at the park for 20 minutes, how far is James from home? How far is James from home after 1 hour?

3. Use the same variables for the time since James has left Tyler's house in seconds, and his overall distance from home in feet. Write an equation that relates these variables for this situation.

4. What is the slope and the *y*-intercept of this function? Explain what each represents in terms of this problem situation.

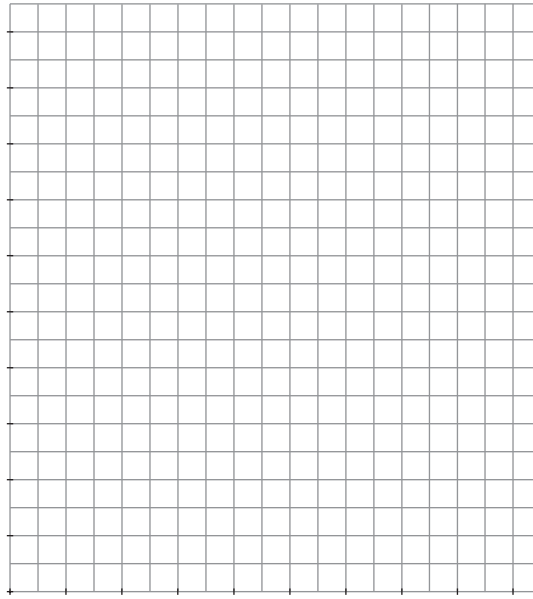> Can I write this equation in slope-intercept form?

5. Complete the table to select appropriate upper and lower bounds to graph this function.

Quantity	Lower Bound	Upper Bound
Time		
Distance		

6. What is the domain of this problem situation? What is the range?

7. Graph the function and label each axis.

8. Describe the graph of this function.

9. Which value does not change but remains constant?

When the *y*-value does not change or remains constant, the function is called a **constant function.**

Problem 3 Heading for Home

After an hour playing basketball, James's brother picks him up and drives him home. It takes James and his brother about 600 seconds to get home.

1. How far is James from home when he leaves the basketball courts?

2. How fast does James's brother drive?

3. Using these two points (when James leaves the basketball courts and when he arrives home), determine the line passing through these points by calculating the slope and the *y*-intercept.

> Will the slope be positive or negative now that he is heading back home?
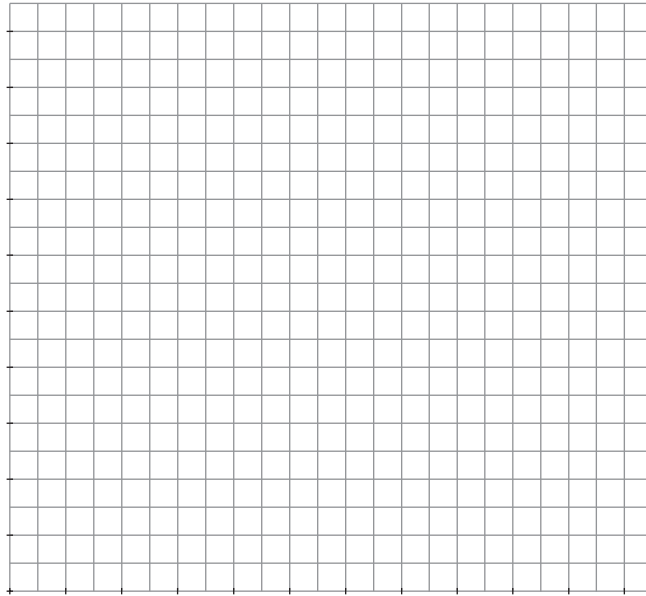
4. What is the slope and the *y*-intercept of this function?

5. Select appropriate upper and lower bounds to graph this function.

Quantity	Lower Bound	Upper Bound
Time		
Distance		

6. What is the domain of this problem situation? What is the range?

7. Graph the function and label each axis.

8. As the time increases, what happens to the distance?

When the value of a dependent variable decreases as the independent variable increases, the function is said to be a **decreasing function**.

Problem 4   **The Whole Journey**

You have just written equations and graphed the separate parts of James's walk to the park from Tyler's house, his time playing basketball at the park, and his car ride home. Let's consider all three parts of his journey away from home as one function and graph.

1. Select appropriate upper and lower bounds to graph his journey as one function on the coordinate plane.

Quantity	Lower Bound	Upper Bound
Time		
Distance		

2. What is the domain of James' journey? The range?

**3.** Graph all three functions as one function and label each axis.

4. Label the parts of the function that are increasing, decreasing, or constant. Explain your reasoning.

How do the rates compare in each segment of the graph?

5. For which values of time is the function increasing?

6. For which values of time is the function decreasing?

7. For which values of time is the function constant?
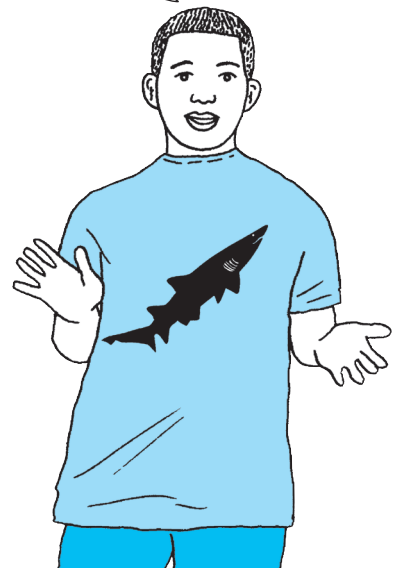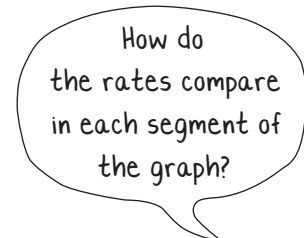
You can describe the intervals of a function by analyzing what happens at specific independent values.

- When a function is increasing for some values of the independent variable, it is said to have an **interval of increase.**
- When a function is decreasing for some values of the independent variable, it is said to have an **interval of decrease.**
- When a function is constant for some values of the independent variable, it is said to have a **constant interval.**

Problem 5   Analyzing Different Functions

1. Complete the table and graph the function $y = |x + 2|$.

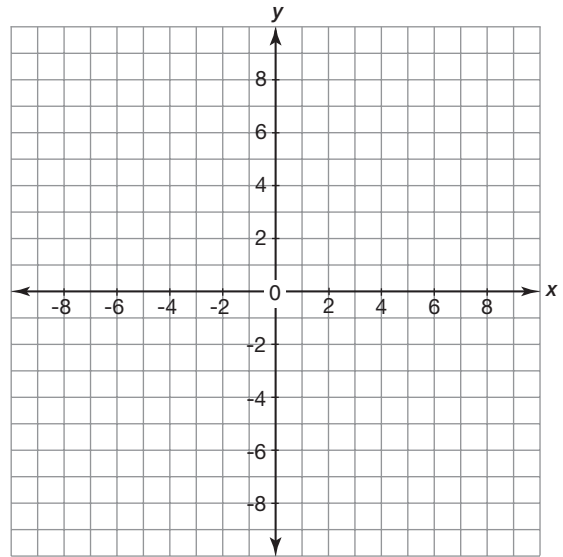| x	y
−6	
−4	
−2	
0	
2	
5	
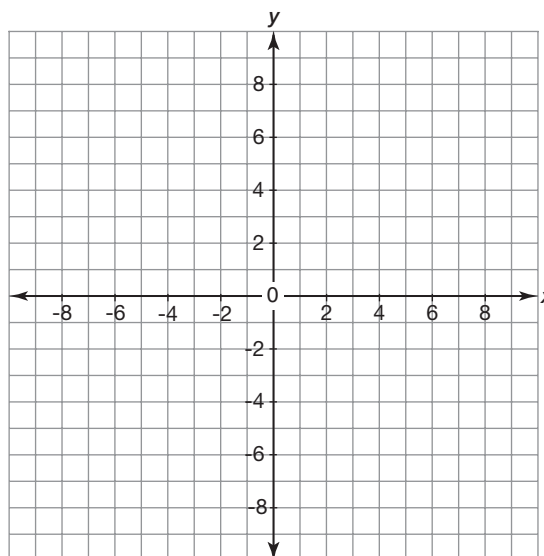
2. Identify the intervals of increase and decrease for this function.

3. Complete the table and graph the function $y = -|x - 2|$.

x	y
−6	
−4	
−2	
0	
2	
4	
5	

4. Identify the intervals of increase and decrease for this function.

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Be prepared to share your solutions and methods.

12.5 PIECEWISE FUNCTIONS

Developing the Graph of a Piecewise Function

Learning Goals

In this lesson, you will:

▶ Develop the graph of a piecewise function from a context with or without a table of values.

▶ Represent a piecewise function algebraically by using appropriate notation, equations, and their domains.

▶ Graph piecewise functions from contexts with or without diagrams.

▶ Physically model the graphs of piecewise functions using technology.

Key Term

▶ piecewise function

The fairy tale *Hansel and Gretel* tells the story of a brother and sister who become lost in the woods and find a house made of candy. As the two children begin eating the candy house, a witch, who lives in the house, captures them and keeps them hostage in order to eat them.

As the story goes, the children left a trail of breadcrumbs from their home and into the woods so that they could find their way back. What similarities and differences are there between this trail of breadcrumbs and the kinds of graphs you have been creating? Create a graph showing a trail of breadcrumbs. How will you label the axes?

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Problem 1   Investigating a Line

1. Suppose you earn $48 helping your neighbor with yard work. You think that you will spend $3 of your earnings each day.

   a. How much money will you have left after 3 days? Show your work.

   b. How much money will you have left after 5 days? Show your work.

   c. How much money will you have left after 10 days? Show your work.

2. Complete the table to show the amounts of money you will have left for different numbers of days.

Time since You Started Spending (days)	Amount of Money Left (dollars)
0	
1	
2	
3	
4	
5	

3. Write an equation that represents the amount of money you have left in terms of the number of days since you started spending your earnings. Be sure to tell what each variable in your equation represents.

4. Use the coordinate plane to create a graph of your equation. Make sure to choose your bounds and intervals first. Label your graph clearly. Do not forget to name your graph. Extend your graph to show when your amount of earnings left would be 0.

Variable Quantity	Lower Bound	Upper Bound	Interval
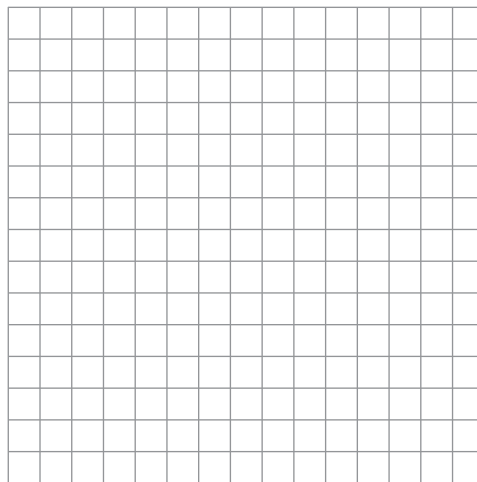
5. Determine the *x*- and *y*-intercepts of the graph. Explain what they mean in terms of the problem situation.

6. Is the slope of your line positive or negative? Does this make sense in terms of the problem situation?
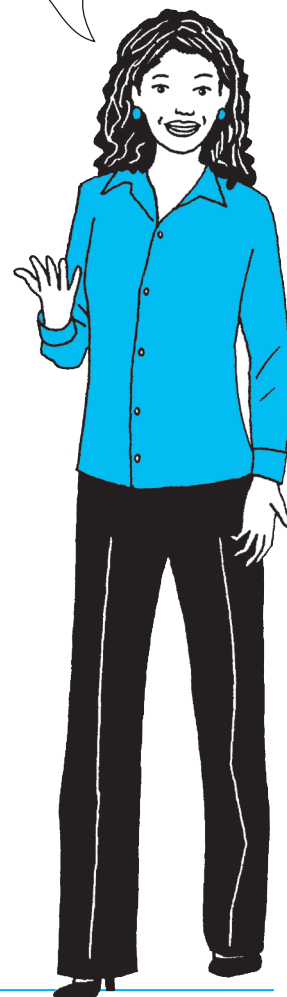
7. Consider your linear function without considering the problem situation. You can determine the domain of your linear function by using your graph.

    a. What is the domain of the function?

> Recall that the set of all possible input values of a function is the domain of the function and the set of all possible output values is the range of the function.

    b. You can also determine the range by using your graph. What is the range of the function?

8. What do you think are the domain and range of any linear function of the form $f(x) = mx + b$? Explain your reasoning.

9. Now consider your linear function again in terms of the problem situation.

   a. What is the domain of the linear function in the problem situation? Explain your reasoning.

   b. What is the range of the linear function in the problem situation? Explain your reasoning.

Problem 2 Representing a Piecewise Function

Suppose that you do not spend the $48 by spending $3 each day. Instead, after 5 days of spending $3 each day, you do not spend anything for 5 days. You then spend $1.50 each day.

1. Complete the table to show the amounts of money you will have left for different numbers of days.

Time Since You Started Spending (days)	Amount of Earnings Left (dollars)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

2. Use the coordinate plane to create a graph from the table of values. Make sure to choose your bounds and intervals first. Label your graph clearly. Extend your graph to show when your amount of earnings left would be 0.

Variable Quantity	Lower Bound	Upper Bound	Interval
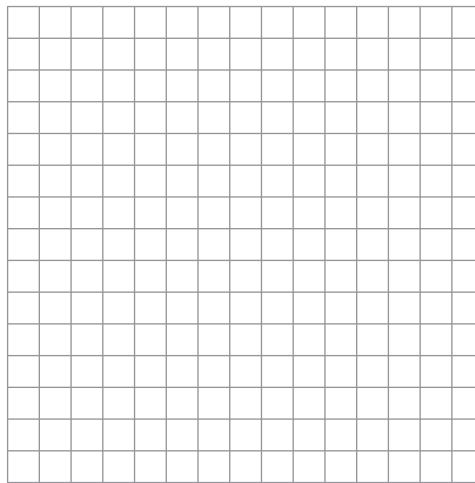
The graph that you created in Question 2 represents a *piecewise function*. A **piecewise function** is a function whose equation changes for different parts, or pieces, of the domain.

3. What is the domain of this function in the problem situation?

4. How many pieces make up this function? (How many different equations are needed to describe this function?)

5. What is the domain of each piece?

6. Write an equation to represent the piece of the function from 0 days to 5 days. Show your work and explain how you determined your answer.

7. Write an equation to represent the piece of the function from 6 days to 10 days. Show your work and explain your reasoning.
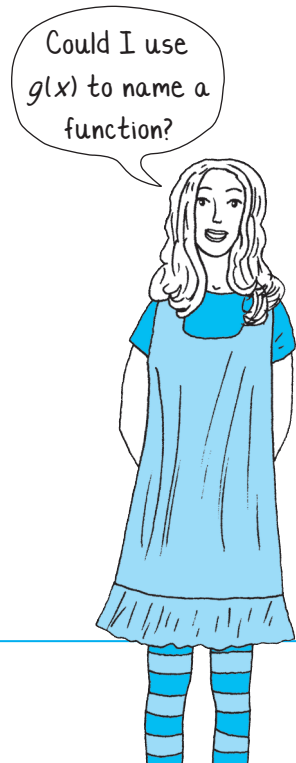
8. Write an equation to represent the piece of the function from 11 days to 32 days. Show your work and explain your reasoning.

Recall, function notation can be used to write functions such that the dependent variable is replaced with the name of the function, such as $f(x)$.

Could I use $g(x)$ to name a function?

9. Complete the definition of your piecewise function, $f(x)$. For each piece of the domain, write the equation you wrote in Questions 6, 7, and 8.

$$f(x) = \begin{cases} \underline{\hspace{3cm}}, & 0 \leq x \leq 5 \\ \underline{\hspace{3cm}}, & 5 < x \leq 10 \\ \underline{\hspace{3cm}}, & 10 < x \leq 32 \end{cases}$$

10. Which piece should you use to determine the *x*-intercept? Which piece should you use to determine the *y*-intercept?

**11.** Determine the intercepts of the graph. Show all your work.

12. When will you run out of money? How does this compare to the number of days it would take you to run out of money in Problem 1?

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Problem 3   Modeling a Piecewise Function

1. Every Tuesday and Thursday, once Kurt gets home from school, he changes his clothes and goes to the community center, which is 1.9 miles from his house, to lift weights. He leaves his house at 3:25 pm and jogs at a steady rate for one mile to his friend Moe's house, which is on the way to the community center. He stops and has a 10-minute break at Moe's house, and then they walk at a consistent pace the rest of the way to the center. They arrive at the center at 4:10 pm.

   a. Draw a graph on the grid provided that could show Kurt's trip from home to the community center. Make sure to label the axes and show the intervals. Do not forget to name your graph.
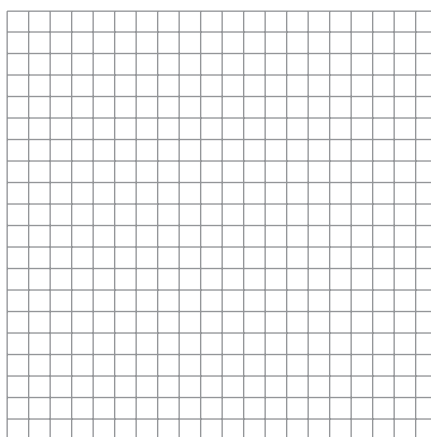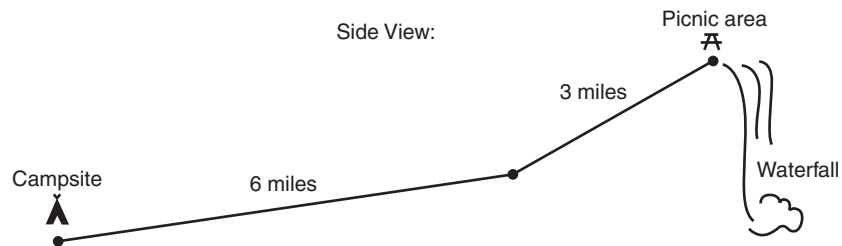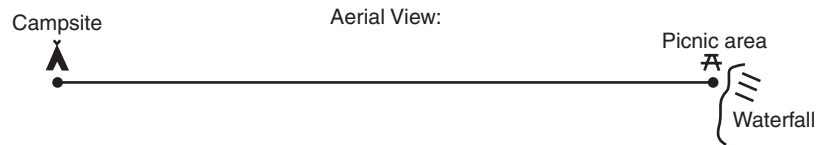
   b. Label the three pieces of the graph *X*, *Y*, and *Z* from left to right.

   c. Order the pieces in terms of their slopes, from least to greatest. Explain why you chose to order the pieces in that matter.

2. Lucy and her friends are hiking from their campsite to a waterfall. They leave their camp at 6:00 AM. They normally hike at a rate of 3 miles per hour, but on steeper parts of the hike, they slow down to 2 miles per hour. They have a one-hour picnic lunch by the waterfall and then reverse their path and hike back to the campsite.

The diagrams shown provide information about the trail that they will be hiking. The diagrams are not drawn to scale.

Aerial View:

Campsite

Picnic area

Waterfall

Side View:

Picnic area

3 miles

Campsite

6 miles

Waterfall

a. Draw a graph modeling Lucy's hiking trip. Label the axes "Time of Day" and "Distance from Campsite." Make sure to show the intervals on the grid and do not forget to name your graph.
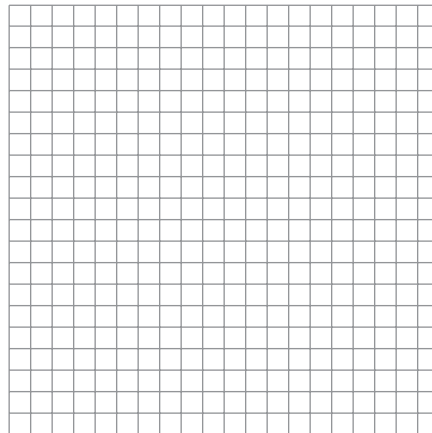
b. Label the pieces of the graph *A*, *B*, *C*, *D*, and *E* from left to right.

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3. Examine your graph.

    a. What pieces have negative slopes? Why are these slopes negative?

    b. Explain the relationship between the slopes of pieces *B* and *D* in terms of the problem situation.

    c. Explain the relationship between the slopes of pieces *A* and *B* in terms of the problem situation.

    d. Why is the slope of piece *A* greater than the slope of piece *B* in the graph, but in the diagram the reverse is true?

    e. Draw a vertical line through the graph to demonstrate its symmetry. Explain why there is symmetry in the graph.

Problem 4   Acting Out a Piecewise Function

You will need a graphing calculator, a Calculator-Based Ranger (CBR), and a connector cable for this activity. You will also need a meter stick and masking tape to mark off distance measures.

Graphs of piecewise functions representing people walking, with time on the *x*-axis and distance on the *y*-axis, are shown after the step-by-step instructions. Your goal is to act out the graph by walking in the way that matches the graph. As you do this, your motion will be plotted alongside the graph to monitor your performance.

Step 1: Prepare the workspace.

- Clear an area at least 1 meter wide and 4 meters long in front of a wall.
- From the wall, measure the distances of 0.5, 1, 1.5, 2, 2.5, 3, 3.5, and 4 meters. Mark these distances on the floor using masking tape.

Step 2: Prepare the technology.

- Connect the CBR to a graphing calculator.
- Transfer the RANGER program from the CBR to the calculator. This only needs to be done the first time. It will then be stored in your calculator.
- Press **2nd** | LINK | ⟶ | ENTER |.
- Open the CBR and press the appropriate button on it for the type of calculator you are using. Your calculator screen will display RECEIVING and then DONE. The CBR will flash a green light and beep.

Step 3: Access the RANGER program.

- Press | PRGM | for program. Choose RANGER. Press | ENTER |.
- Press | ENTER | to display the MAIN MENU.
- Choose | APPLICATIONS |. Choose | METERS |.
- Choose | MATCH | or | DISTANCE | MATCH |.
- Press | ENTER |. A graph will be displayed.

Step 4: Act out the graph.

- Examine the graph. Plan your path. Use the scale to gauge where to begin in relation to the wall. Will you walk toward or away from the wall? Will you walk fast or slow?

- Hold the graphing calculator in one hand and the RANGER in the other hand. The lid of the RANGER should be aimed toward the wall.

- Press $\boxed{\textbf{ENTER}}$. Begin walking in a manner that matches the graph. Use the scale and floor markings as guides. You will hear a clicking sound and see a green light as your motion is plotted alongside the piecewise graph on the graphing calculator.

- When the time is finished, examine your performance. What changes should you make?

- Press $\boxed{\textbf{ENTER}}$ to display the OPTIONS menu. Choose $\boxed{\textbf{SAME MATCH}}$.

- Press $\boxed{\textbf{ENTER}}$ and try the walk a second time.

- Continue acting out walks by pressing $\boxed{\textbf{ENTER}}$ and **NEW MATCH**.

- When finished, press $\boxed{\textbf{ENTER}}$, choose $\boxed{\textbf{MAIN MENU}}$, and $\boxed{\textbf{QUIT}}$.
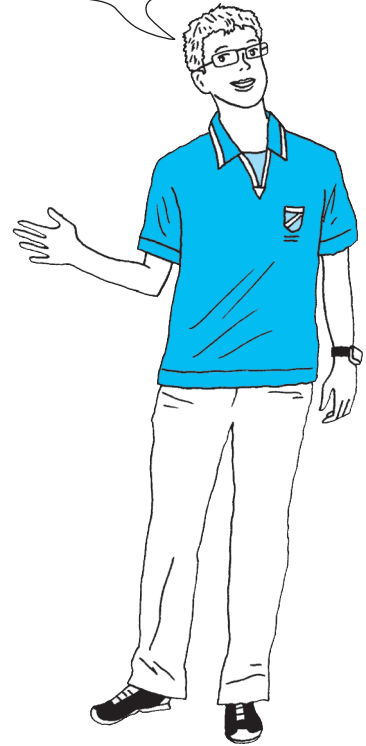
12
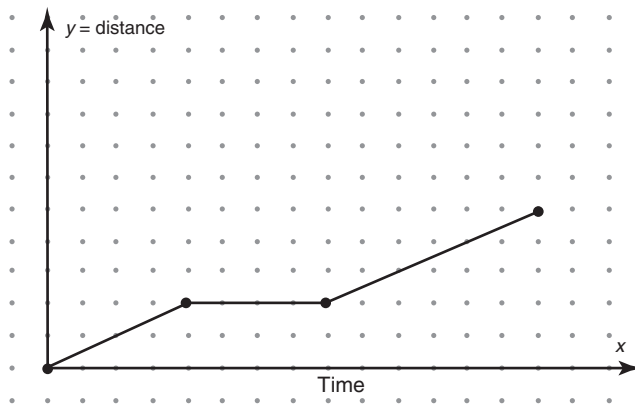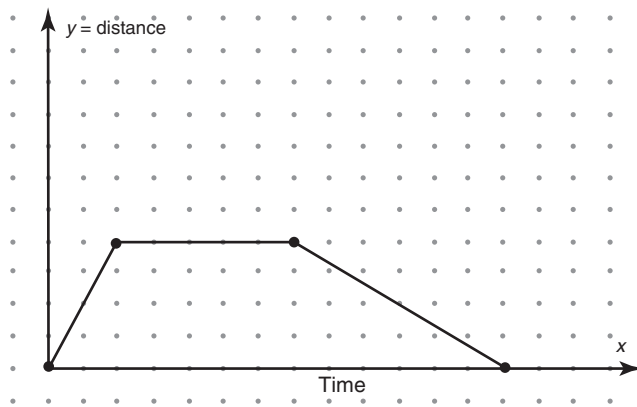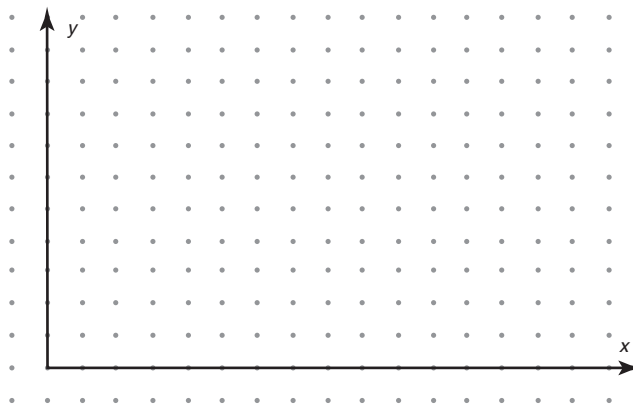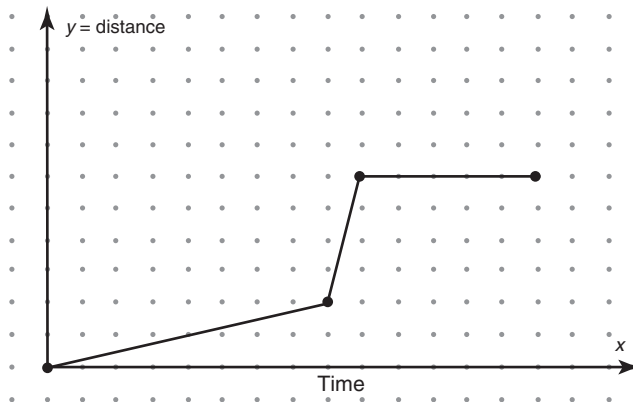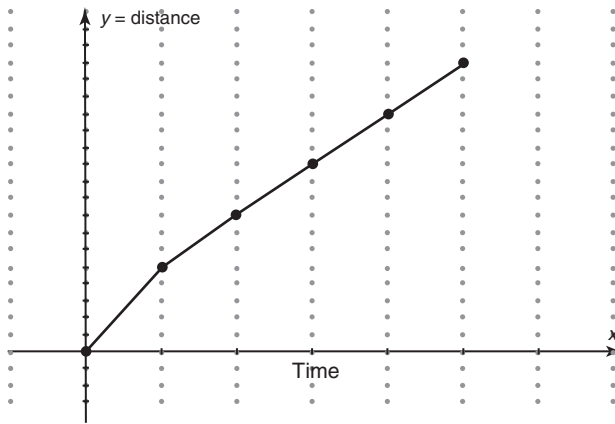
1. How did you decide where to stand when beginning to act out a graph?

Did this activity help you make sense of the graphs?

2. How did you decide when to walk toward the wall and when to back up from the wall?

3. How did you act out a horizontal segment?

4. How did you decide how fast to walk?

Be prepared to share your solutions and methods.

Chapter 12　Summary

Key Terms

▶ increasing function (12.4)
▶ constant function (12.4)
▶ decreasing function (12.4)
▶ interval of increase (12.4)

▶ interval of decrease (12.4)
▶ constant interval (12.4)
▶ piecewise function (12.5)

12.1　Analyzing Problem Situations Using Multiple Representations

A problem situation can be represented multiple ways.

Example

Natalie is biking around a small park. The table shows the time it took Natalie to complete one lap around the park on the 1.8 mile bike path.

Time (minutes)	Distance (miles)
0	0
10	1.8

The calculation of Natalie's average speed in miles per minute is shown.

$$\text{Average speed} = \frac{1.8 \text{ miles}}{10 \text{ minutes}}$$
$$= 0.18 \text{ mile per minute}$$

Natalie's average speed is 0.18 mile per minute.

Next, an equation is shown to represent the relationship between the time and distance Natalie bikes. The first step is to define the variables and what they represent.

Let t represent to the time Natalie bikes.
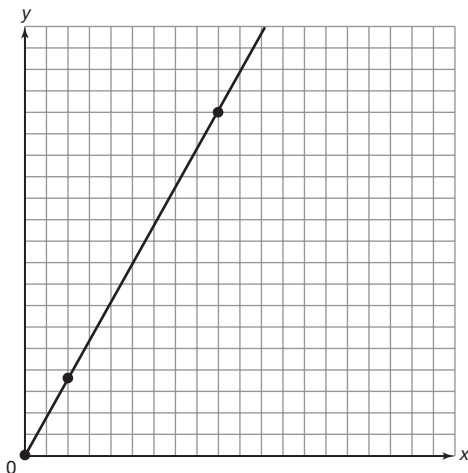
Let d represent to the distance Natalie bikes.

$d = 0.18t$

Once the variables are defined and an equation is determined, substitution can be used to determine how far Natalie can ride in 45 minutes.

$$d = 0.18t$$
$$d = 0.18(45)$$
$$d = 8.1$$

If Natalie bikes for 45 minutes, she will bike 8.1 miles.

Finally, a graph shows the relationship between the time and distance Natalie biked.

12.2 Interpreting the Standard Form of a Linear Equation

A problem situation can be represented with the standard form of a linear equation.

Example

Maria is baking muffins from scratch for the school bake sale. It takes Maria 1.25 hours to bake each batch of muffins.

An expression can be written to represent the amount of time it takes Maria to bake a certain number of batches of muffins.

Let x equal the number of muffin batches Maria bakes.
The expression $1.25x$ equals the amount of time it takes Maria to bake x batches of muffins.

Once a variable is defined and an expression is determined, it can be determined how long it takes Maria to bake batches of muffins. For example, substitution is used to determine how long it takes Maria to bake 3 batches of muffins.

$$1.25x = 1.25(3)$$
$$= 3.75$$

It would take Maria 3.75 hours to bake 3 batches of muffins.

Isabel is baking pre-made frozen muffins for the school bake sale. It takes Isabel 0.75 hour to bake each batch of muffins.

As with Maria, an expression can be written to represent the amount of time it takes Isabel to bake a certain number of batches of muffins.

Let y equal the number of muffin batches Isabel bakes.

The expression $0.75y$ equals the amount of time it takes Isabel to bake y batches of muffins.

Once the variable is defined and an expression is determined, substitution will determine how many batches of muffins Isabel can bake in a certain amount of hours.

$$0.75y = 3$$
$$\frac{0.75y}{0.75} = \frac{3}{0.75}$$
$$y = 4$$

Isabel could bake 4 batches of muffins in 3 hours.

Supposed Maria and Isabel had to take turns sharing the school kitchen to bake the muffins. The school kitchen is available for 10 hours. An equation in standard form can be written to represent this situation.

$$1.25x + 0.75y = 10$$

Using the equation and substitution, it can be determined how many batches of muffins Isabel can bake when Maria bakes 4 batches.

$$1.25x + 0.75y = 10$$
$$1.25(4) + 0.75y = 10$$
$$5 + 0.75y = 10$$
$$0.75y = 5$$
$$y \approx 6.7$$

If Maria bakes 4 batches of muffins, Isabel can bake 6 batches of muffins in the time allotted.

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12.3 **Determining the Slope and Intercepts of a Linear Equation in Standard Form**

The standard form of a linear equation can be used to determine the *x*-intercept, the *y*-intercept, and the slope of the graph of the linear equation.

Example

Use the standard form of the linear equation to determine the *x*-intercept, the *y*-intercept, and the slope of the graph of the linear equation.

$$2x + 3y = 18$$

To determine the *y*-intercept, solve the equation when $x = 0$.

$$2x + 3y = 18$$
$$2(0) + 3y = 18$$
$$3y = 18$$
$$y = 6$$

The *y*-intercept is (0, 6).

To determine the *x*-intercept, solve the equation when $y = 0$.

$$2x + 3y = 18$$
$$2x + 3(0) = 18$$
$$2x = 18$$
$$x = 9$$

The *x*-intercept is (9, 0).

The slope can be calculated from the intercepts.

$$m = \frac{0 - 6}{9 - 0}$$
$$= \frac{-6}{9}$$
$$= -\frac{2}{3}$$

The slope is $-\frac{2}{3}$.

12.3 Converting Linear Equations between Standard Form to Slope-Intercept Form

To convert a linear equation in standard form to slope-intercept form, solve for y. To convert a linear equation in slope-intercept form to standard form, solve for the constant term.

Example

To convert the linear equation $3x + 4y = 48$ to slope-intercept form, follow the steps shown.

$$3x + 4y = 48$$
$$4y = -3x + 48$$
$$y = -\frac{3}{4}x + 12$$

To convert the linear equation $y = \frac{3}{5}x + 2$ to standard form, follow the steps shown.

$$y = \frac{3}{5}x + 2$$
$$5y = 3x + 10$$
$$-3x + 5y = 10$$

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Identifying Functions as Increasing, Decreasing, or Constant

When both the value of the independent variable and the value of the dependent variable of a function increase, the function is said to be an increasing function. When the value of the dependent variable does not change or remains constant as the value of the independent variable increases, the function is called a constant function. When the value of the dependent variable decreases as the value of the independent variable increases, the function is said to be a decreasing function.
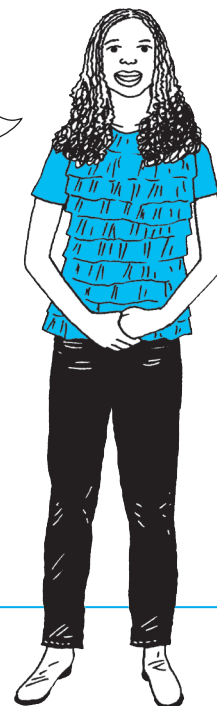
Example
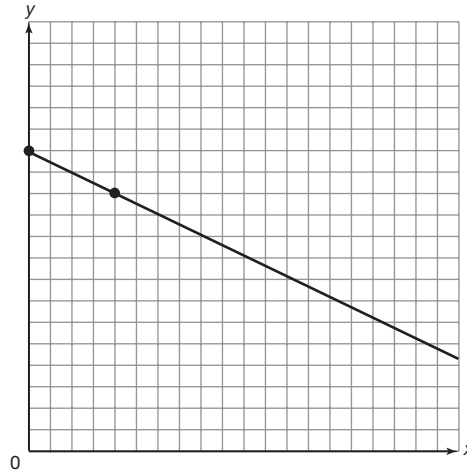
The graph of $y = 3x + 1$ is shown.
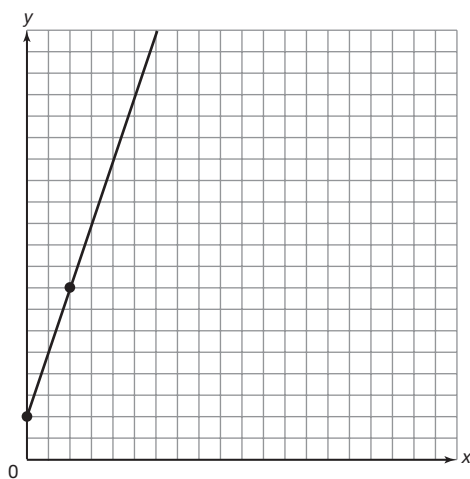
The independent variable increases, the dependent variable increases. The function is an increasing function.

> Mental activities, like puzzles and even math problems, can improve and increase your brain function. You wouldn't want your brain function to be constant or decreasing!
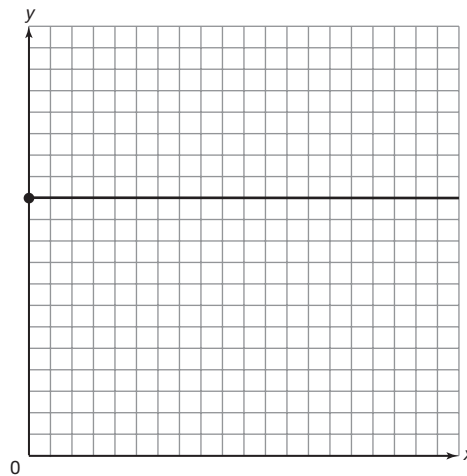
The graph of $y = -\frac{1}{2}x + 7$ is shown.

The independent variable increases, the dependent variable decreases. The function is a decreasing function.

The graph of $y = 6$ is shown.

The independent variable increases, the dependent does not change or remains constant. The function is a constant function.

Identifying Intervals of Increase, Decrease, and Constant Values of a Function

When a function is increasing for some values of the independent variable, it is said to have an interval of increase. When a function is decreasing for some values of the independent variable, it is said to have an interval of decrease. When a function is constant for some values of the independent variable, it is said to have a constant interval.

Example

The graph of $y = |x + 3|$ is shown.
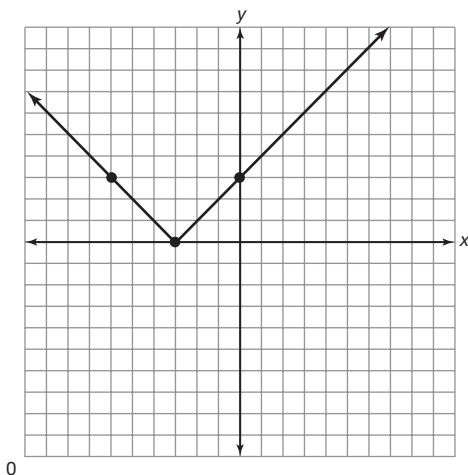
The function decreases until -3 and then increases after -3.

12.5 Writing a Piecewise Function from a Table or Context

Piecewise functions can be used to model any situation with varying rates of change. A piecewise function represents more than one function, each of which corresponds to a part of the domain. Use the slope and a point from a problem situation to determine each part of the function. Write the function using x to represent a number from the domain of the function f. Use the left brace symbol "{" to show that all expressions in the group are part of the function.

Example

Hour	Toys Painted
0	0
1	6
3	18
4	24
5	24
6	29
9	44

$$f(x) = \begin{cases} 6x, & 0 \leq x \leq 4 \\ 24, & 4 < x \leq 5 \\ 5x - 1, & 5 < x \leq 9 \end{cases}$$

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Graphing a Piecewise Function from a Table, Context, or Diagram

The graph of a piecewise function may consist of a series of line segments with different slopes. It can be generated from a table, an algebraic representation, or a context with or without a diagram. Graph each section of the piecewise function on the same graph with the appropriate bounds and intervals.

Example

Salvador starts painting toys at 8:00 AM. He steadily paints 6 toys per hour until his lunch break at 12:00 PM. He gets back to work an hour later and paints at a rate of 5 toys per hour until his shift is over at 5:00 PM.